



Arithmetic 3

Number Representation

By Robin Bacchus

Writing or showing numbers

- Counting out loud is verbal and momentary and not visual.
- Often we want to show a number visually or tactilely
- For small numbers, we can count on the fingers of both hands.
- If each digit (finger or thumb) represents 'one', we can get to 10.



Counting further...

- If we differentiate between fingers and thumbs we can go further.
- However, if we let a finger 'f' on our right hand represent 'one' and the thumb 't' represent 'five' we can get on one hand to: $tffff = 9$.
- Now add the left hand where, now, the fingers 'F' = 10, and the thumb 'T' = 50, we can get to $TFFFF = 90$. Both hands together give us $TFFFFtffff = 99$, that is:
- $50+10+10+10+10+5+1+1+1+1 = 99$
- which is nearly 10 times more than when all the digits have the same value (1).

Counting Device: Abacus

Historical Significance

The abacus is one of the earliest tools used for counting and arithmetic, dating back thousands of years in central and eastern Asia.

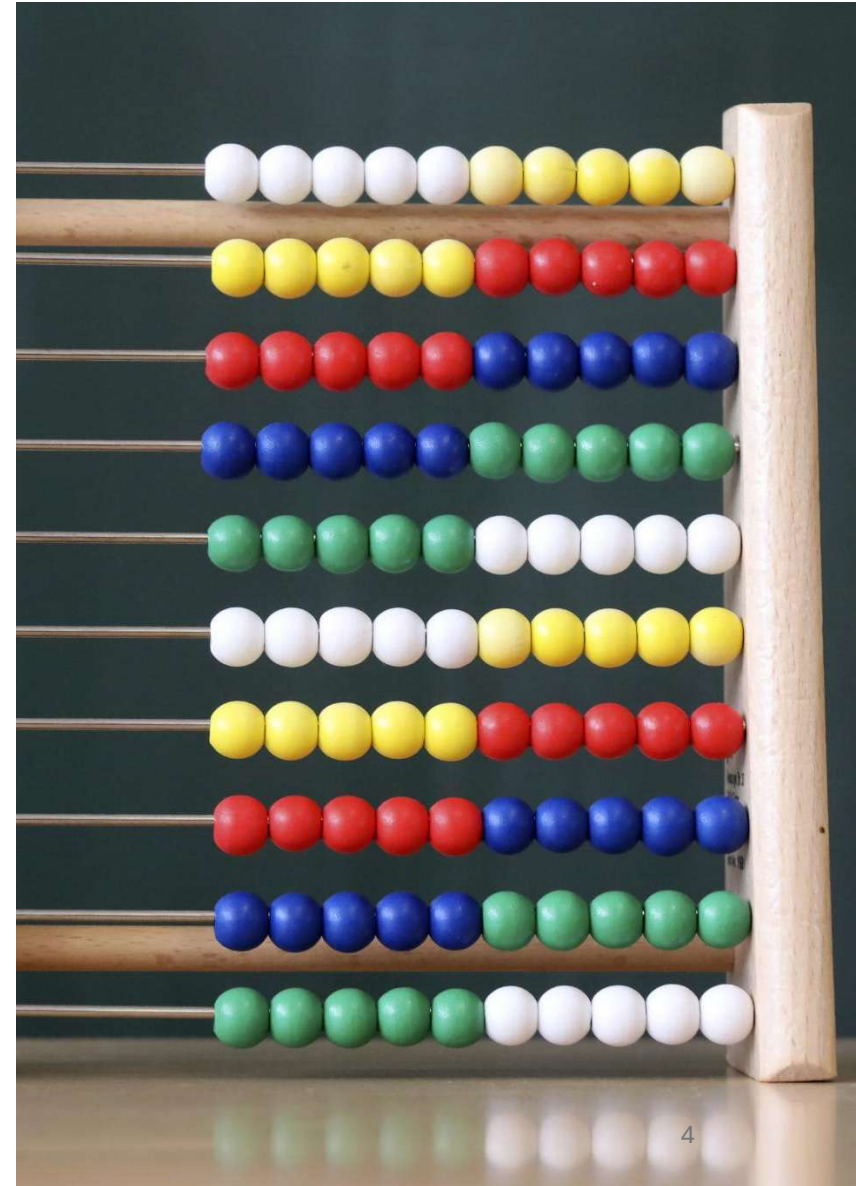
Design & Functionality

Comprising rods with beads that can be moved, the abacus helps users perform calculations visually and physically, enhancing understanding of numerical concepts.

Modern Relevance

Though largely replaced by digital calculators, the abacus remains a valuable educational tool for teaching basic math skills and cognitive development. The Russian one shown has ten rows of ten – 2 groups of 5 - sliding beads.

Number Representation

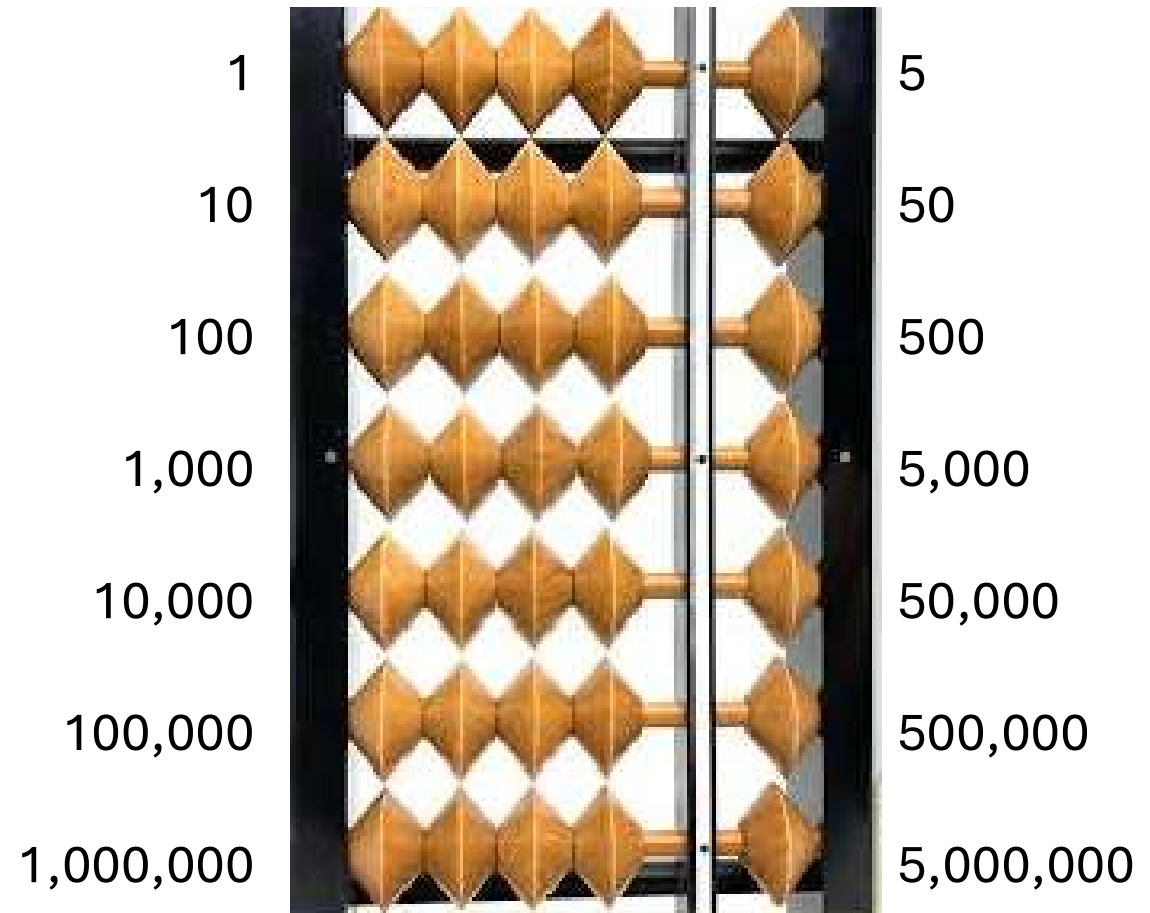


Counting Devices – the Abacus

COMPONENT	DESCRIPTION	MATERIAL	FUNCTION
Frame	Structure that holds beads	Wood or metal	Supports beads and structure
Beads	Movable counters used for counting	Wood, plastic, or metal	Represent quantities for calculations
Rods	Horizontal or vertical bars for beads to slide on	Wood or metal	Hold beads in place and allow movement
Counting Technique	Method of using beads to count or calculate	Manual manipulation of beads	Tactile method for counting and arithmetic

The Japanese Soroban positioned horizontally

- The Soroban has a central “value” bar with four sliders on the left side and one on the right.
- You only count the sliders against the central bar.
- The one to the right can show all the numbers up to ten million.
- It can be used for the four basic Arithmetic operations: +, -, \times , /





A modern Japanese Soroban.

The visible part of the soroban represents the number 1234567890, each column indicating one digit, with the lower beads representing "ones" and the upper beads "fives".

Use of Soroban in Japanese schools

- The soroban is still taught in some primary schools as a way to visualize and grapple with mathematical concepts.
- The practice of soroban includes the teacher reciting a string of numbers (addition, subtraction, multiplication, and division) in a song-like manner where at the end, the answer is given by the teacher.
- This helps train the ability to follow the tempo given by the teacher while remaining calm and accurate.
- In this way, it reflects on a fundamental aspect of Japanese culture of practicing meditative repetition in every aspect of life.

Simple early writing of numbers: Tallying as you count

#	English	Marks	Tallying	Roman
1	one	*		I
2	two	**		II
3	three	***		III
4	four	****		IV
5	five	*****	 	V
6	six	*****	 	VI
7	seven	*****	 	VII
8	eight	*****	 	VIII
9	nine	*****	 	IX
10	ten	*****	 	X

Number Representation

Tallying – making marks while you count things

- Each row shows the symbol from the previous row with a mark **added**.
- Each column shows symbols becoming more **condensed**.
Note how the Tallying result for 5 looks like a hand with the thumb laid across the fingers, which becomes the V of Roman numerals. Then the two Vs become X (one up and the other down!) to make ten.
- However, the later **Roman numerals** introduce a new element: **If** a smaller symbol is placed after a larger symbol ($VI = 5+1 = 6$), **then** it is **added**;
But **if** it is placed in front ($IV = 5-1 = 4$), **then** it is **subtracted**, so **position** becomes important.

Roman Numerals – no zero

- The Roman system is similar to the Japanese Soroban using decimal multiplies of 1 and 5, but did not go to very large numbers. 100 was **Centum**, 1,000 was **Mille**.

Equivalentents			
1	I	V	5
10	X	L	50
100	C	D	500
1,000	M		

- There was no *zero* because no-one found it useful to count nothing.
- Larger numbers were later written by using a **vinculum**, that is a bar written over a number to multiply it by 1,000; a double bar by 1,000,000. So 5,000 is \overline{V} , and 10,000 is $\overline{\overline{X}}$

Our modern numbering system

- Our modern numbering system, known as the [Hindu-Arabic numeral system](#), originated in **India** by the 8th century AD, featuring 10 unique symbols (0-9) and a place-value system.
- It was adopted and refined by Arab mathematicians—notably **al-Khwarizmi**—before spreading to Europe, eventually replacing inefficient Roman numerals over several centuries.
- **The Power of Zero:** The critical development of the symbol for zero ("0") to denote ***empty place*** values was formalized by Indian mathematicians like **Brahmagupta** in the 7th century, distinguishing numbers like 408 from 48.

End of the Roman System

- **Arab Transmission:** Arabic scholars in the 10th century, notably in Spain and North Africa, adopted these numerals and spread them westward.
- **European Adoption:** In 1202, Leonardo **Fibonacci** of Pisa introduced these numbers to Europe in his book *Liber Abaci*, [Book of the Abacus] training merchants on their efficiency.
- **Modernization:** It took until the 14th–15th centuries (beginning of the 5th Post-Atlantic Age or Age of Reason) for these Hindu-Arabic numbers to fully replace Roman numerals in Europe.

Why it Caught On:

- Unlike the Roman system (I, II, III, IV, V, etc.) or the Egyptian numerals, the **Hindu-Arabic** system allowed for complex **calculation**, **fractions** [decimals], and **massive** numbers using only 10 symbols through place value (ones, tens, hundreds, etc.).

The symbols used for numbers

- The symbols we use for numbers need to be:
 1. Easy and quick to write.
 2. Easy to recognise.
 3. Sufficiently different from other symbols.
(this is not quite the case as
O and 0
l and 1
are very similar)
- What is the story behind their creation?

Creation –
one possible
origin

Hindu-Arabic numeration system



2 angles

7 angles

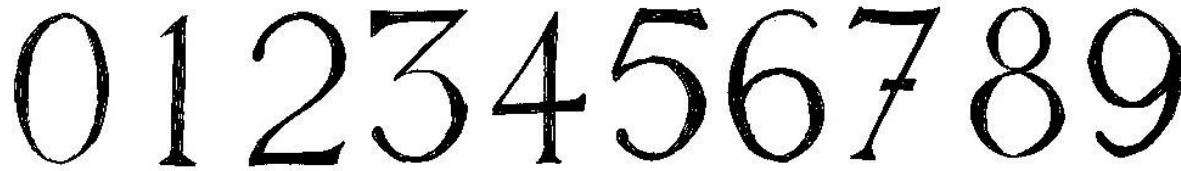
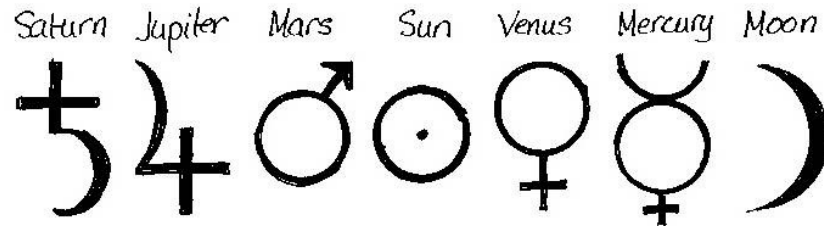
Another connection – symbols for planets

- In earlier times the visible “wandering stars” of the zodiac – the planets – were named after gods. They were considered to be many more than just a single ‘supreme deity’.
- Today those gods are largely forgotten and the planets are deemed to be large spherical rocks or gas clouds orbiting the sun along with the Earth.
- However the planets, along with the sun and moon, were represented by symbols which have some resemblance to the Hindu-Arabic numerals we use today.

Outermost planets

Inner planets

Planetary symbols >>>



Zero One Two Three Four Five Six Seven Eight Nine
Circle Line
Curved Straight
Will Think
Empty All

The Arabic origin numeric symbols used today have a connection to the Chaldean symbols denoting the 5 visible planets, plus sun and moon.

Schema huius præmissæ diuisionis Sphærarum .

Ptolemaic Geocentric Schema:

- *Heaven
- *Prime mover
- *Zodiac stars
 - = 12 constellations
- *7 Planets
- *Earth
 - = 4 elements

