Fibonacci Spiral and the SUNFLOWER

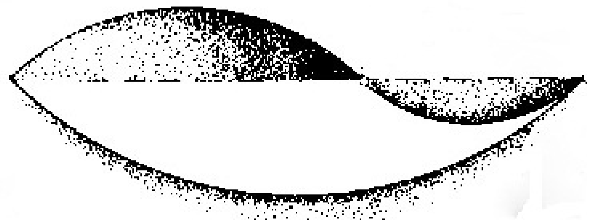
By Robin Bacchus

Fibonacci

- Leonardo Bonacci [good nature] (c. 1170 – c. 1240–50), was an Italian mathematician from the Republic of Pisa, considered to be "the most talented Western mathematician of the Middle Ages".
- The name he is commonly called, ***Fibonacci***, is short for *filius Bonacci* ('son of Bonacci').
- Fibonacci popularized the Indo–Arabic numeral system in the Western world replacing the Roman numerals and introducing the concept zero, 0, or nothing, a placeholder, primarily through his composition in 1202 of *Liber Abaci* (*Book of Calculation*) using the abacus. It allowed easy calculation using a place-value system. He also introduced Europe to the sequence of Fibonacci numbers, which he used as an example in *Liber Abaci*.

Golden Ratio
or: Divine Proportion :

- When a length of a line is divided by a point into two parts, and the ratio (division) of the shorter part to the longer part is the same as the ratio of the longer part to the whole then it is divided in the:
  = GOLDEN RATIO =

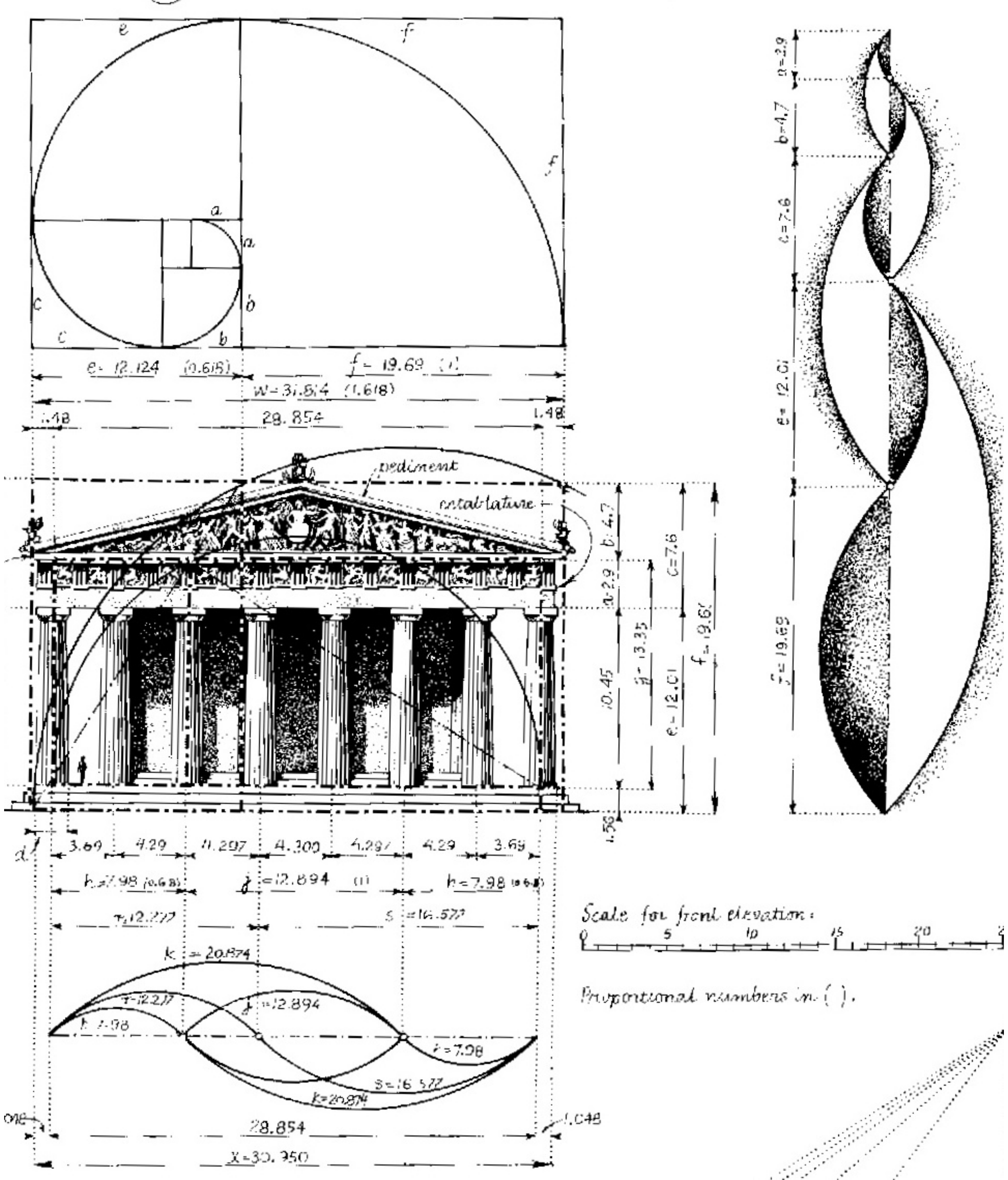


Golden Ratio' or 'Divine Proportion'

- In mathematics, two quantities are in the golden ratio if their ratio is the same as the ratio of their sum to the larger of the two quantities. Sum/Larger = Larger to Smaller
- Expressed algebraically, for quantities **a** [larger] and **b** [smaller] with $a > b > 0$
- $(a+b)/a = a/b = \phi$ [Greek letter 'phi' and/or Latin letter 'G' or 'g' as a symbol for the Golden Ratio.]

Greek temple

in golden ratio proportions

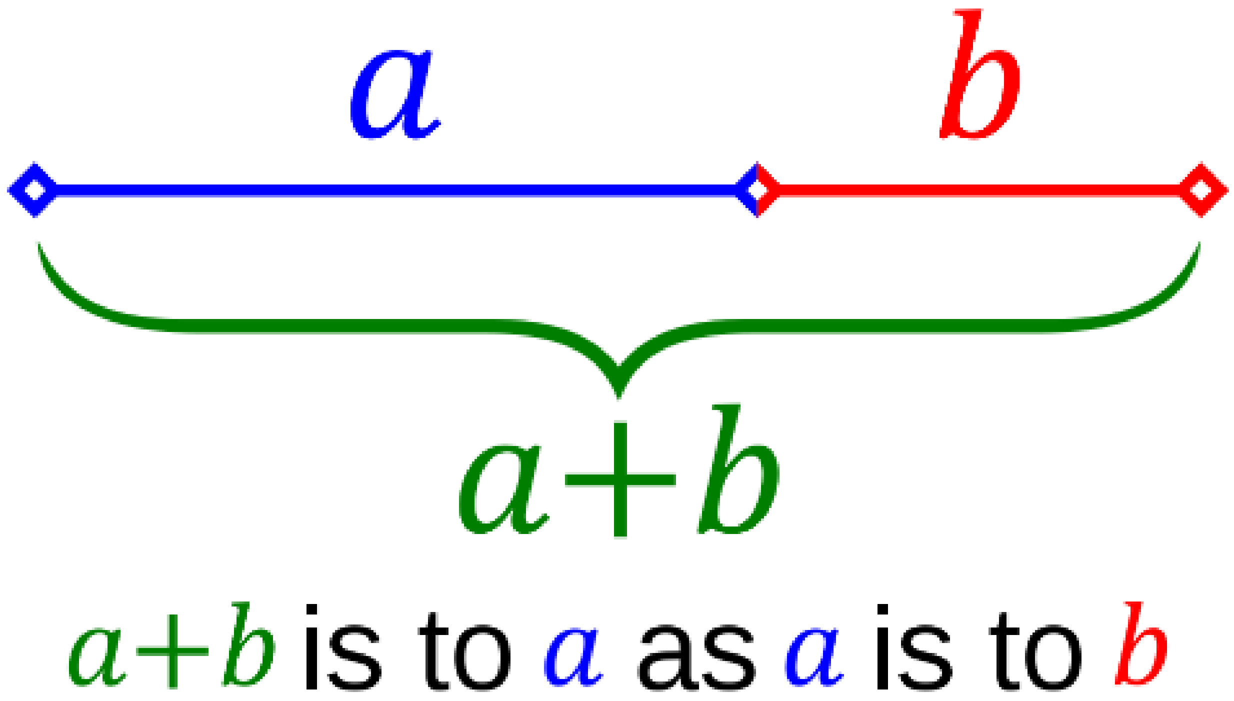


Fibonacci
= “Son of Good Nature” =

- What is the secret of this series of numbers?
- 1,1,2,3,5,8,13,21,34,55,..................

- 1+2=3
- 2+3=5
- 3+5=8
- 5+8=13
- 8+13=21
- 13+21=34
- Smaller + Larger = Whole

The figure below illustrates the geometric relationship.



Also, this can be expressed as:

(a+b)/a = a/b   [multiply each element by ‘ab’]

➔ : (a+b)*b = a*a   [expand]

➔ : a*b + b*b = a*a   [collect elements on LHS}

➔ : a^2 -a.b – b^2 = 0   [divide all elements by b^2]

➔: $(a/b)^2$ – (a/b) – 1 = 0   [(a/b) is the Golden ratio, G]

➔: G^2 – G – 1 = 0.

This is a quadratic equation whose solution is

$G = (1 + \sqrt{5})/2 = 1.618,033,988,7....$

Note that $\sqrt{5}$ is part of the equation, and can be derived from a right-angled triangle by Pythagoras' theorem.

The Fibonacci Series

- The Fibonacci Series is a series of numbers where any element is the sum of the previous two elements.

- $F(n+1) = F(n) + F(n-1)$

Two examples with very different first pair.

After 11 steps the two ratios for G are very nearly equal at 1.617….

algebra	Example 1	Ratio 1	Example 2	Ratio 2	
a	**1**		**100**		
b	**1**	1/1=1	**1**	1/100=0.01	lo
a+b	2	2/1=2	101	101/1=101	hi
a+2b	3	3/2=1.5	102	102/101=1.009,9…	lo
2a+3b	5	5/3=1.666…	203	203/102=1.990,196…	hi
3a+5b	8	8/5=1.6	305	305/203=1.502,463…	lo
5a+8b	13	13/8=1.625	508	508/305=1.665,574…	hi
8a+13b	21	21/13=1.615,384..	813	813/508=1.600,398…	lo
13a+21b	34	34/21=1.619,047..	1,321	1,321/813=1.624,846…	hi
21a+34b	55	55/34=1.617,647..	2,134	2134/1321=1.615,443…	lo
34a+55b	89	89/55=1.618,182..	3,455	3455/2134=1.619,025…	hi
55a+89b	144	144/89=1.617,978..	5,589	5589/3455=1.617,656…	lo

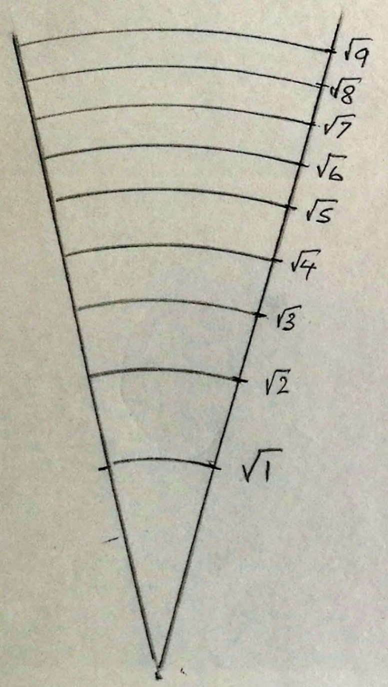
Sunflower seed pattern

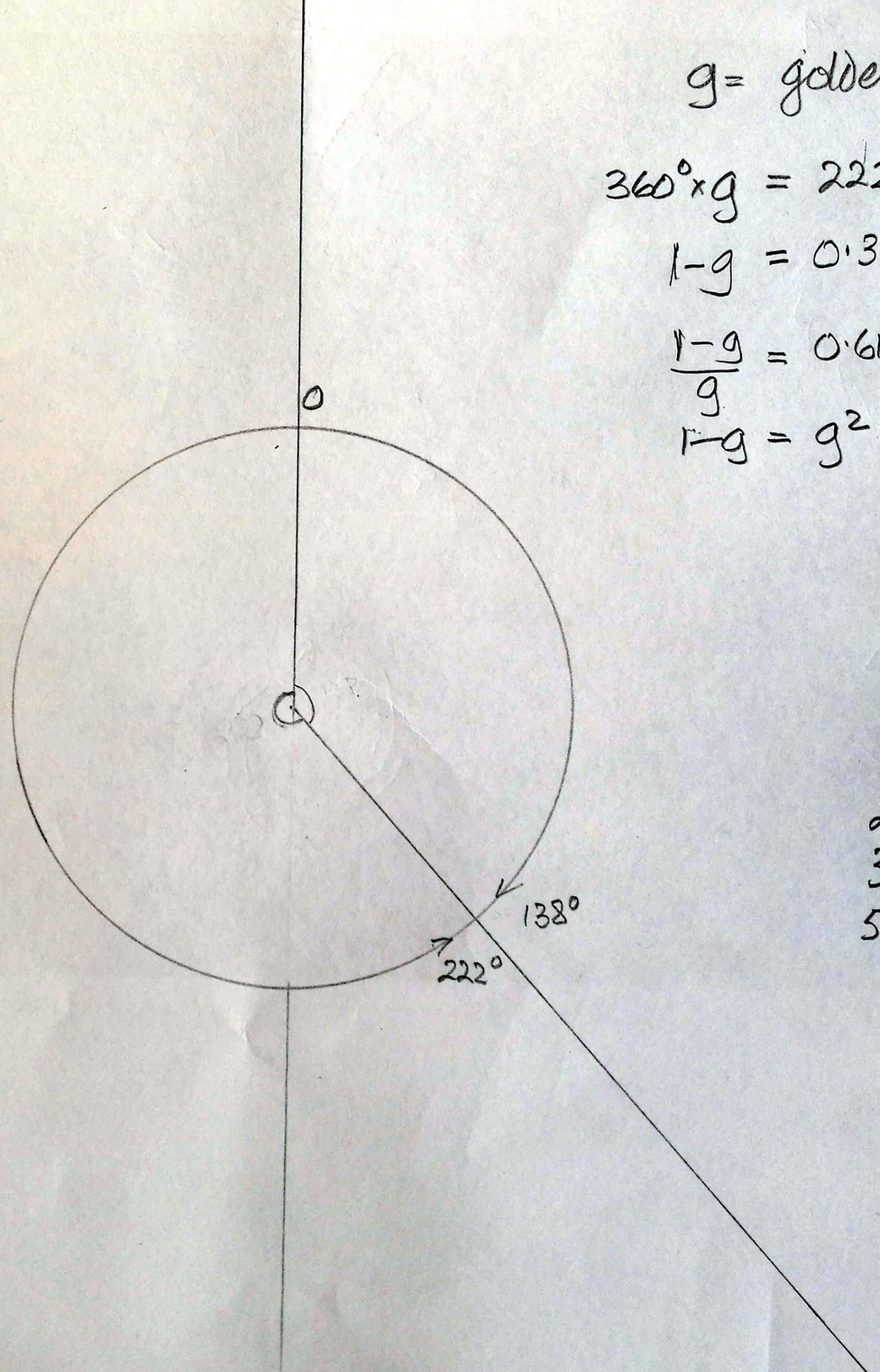
- When looking at the seed pattern we can see spirals going both clockwise and anticlockwise.
- Each seed is a similar size whether nearer the centre or nearer the periphery.
- When we count the number of spirals, side-by-side in a pattern we can numbers like 21, 34, 55, 89 which are all numbers found in the Fibonacci sequence.

Geometry of the spirals

- The leading idea is that each has a similar area around it.
- Thus in a sector of a circle the width increases with the radius.
- But the thickness of each part is inverse to the radius.
- In the second diagram the 360° is divided in the Golden ratio: 222° to 138°

Equal areas





g = golden ratio = 0.618...

$360° \times g = 222°$

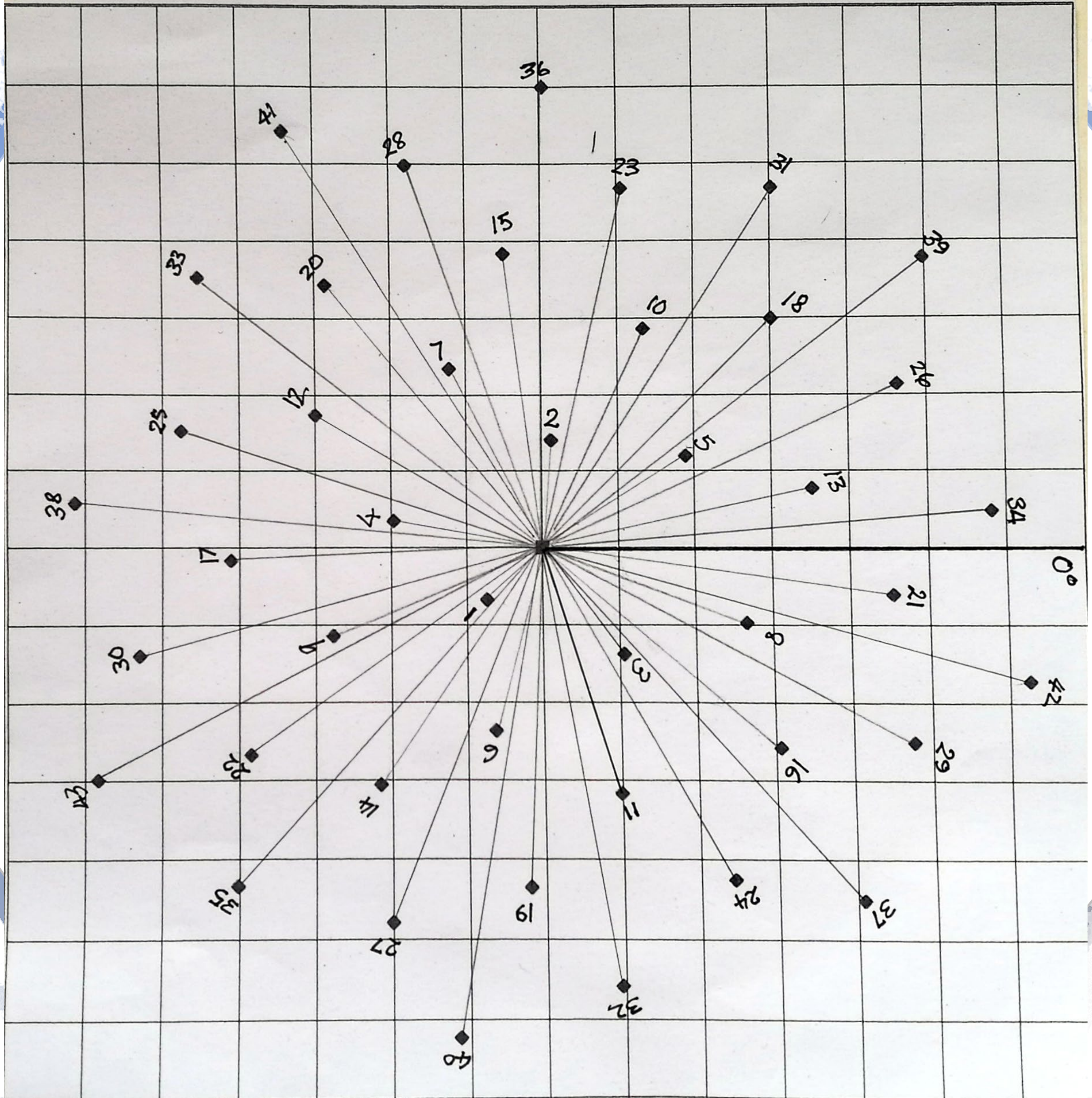
$1 - g = 0.382...$

$\frac{1-g}{g} = 0.618 = g$

$1 - g = g^2$

1 > 1/2 = 0.500
2 > 2/3 = 0.666
3 > 3/5 = 0.600
5 > 5/8 = 0.625
8 > 8/13 = 0.6154
13 > 13/21 = 0.6190
21 > 21/34 = 0.6176
34 > 34/55 = 0.6182
55

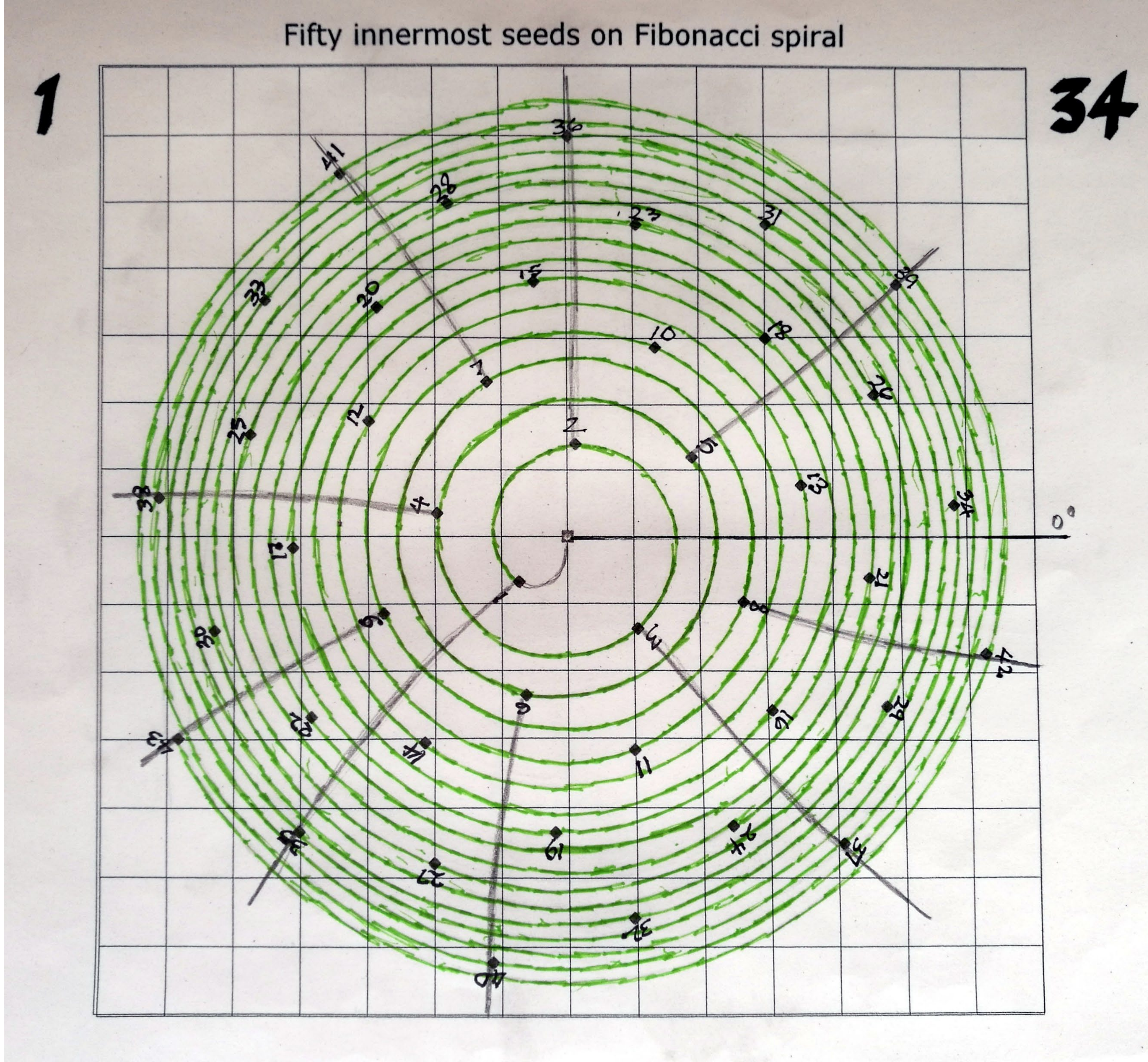
Fifty innermost seeds on Fibonacci spiral



The Fibonacci Sequence on Fermat’s Spiral.
Angle = 360°/(1-G) = 138° gives the mathematical/geometrical centre of the seeds in a sunflower head.
Note how the spacing between successive spirals gets gradually smaller – inverse to the radius.
The area between the spirals between successive remains constant: that is Natura has arranged that **each seed** has the same space wherever it is! That is *magic*.

The space between spirals is inversely proportional to the radial distance from the centre.
Th one Green clockwise spiral (1) connects every seed.
The 34 Black, almost radial, anticlockwise spirals, connect every 34^{th} seed.

(The Fibonacci series goes: **1** 2 3 5 8 13 21 **34)**

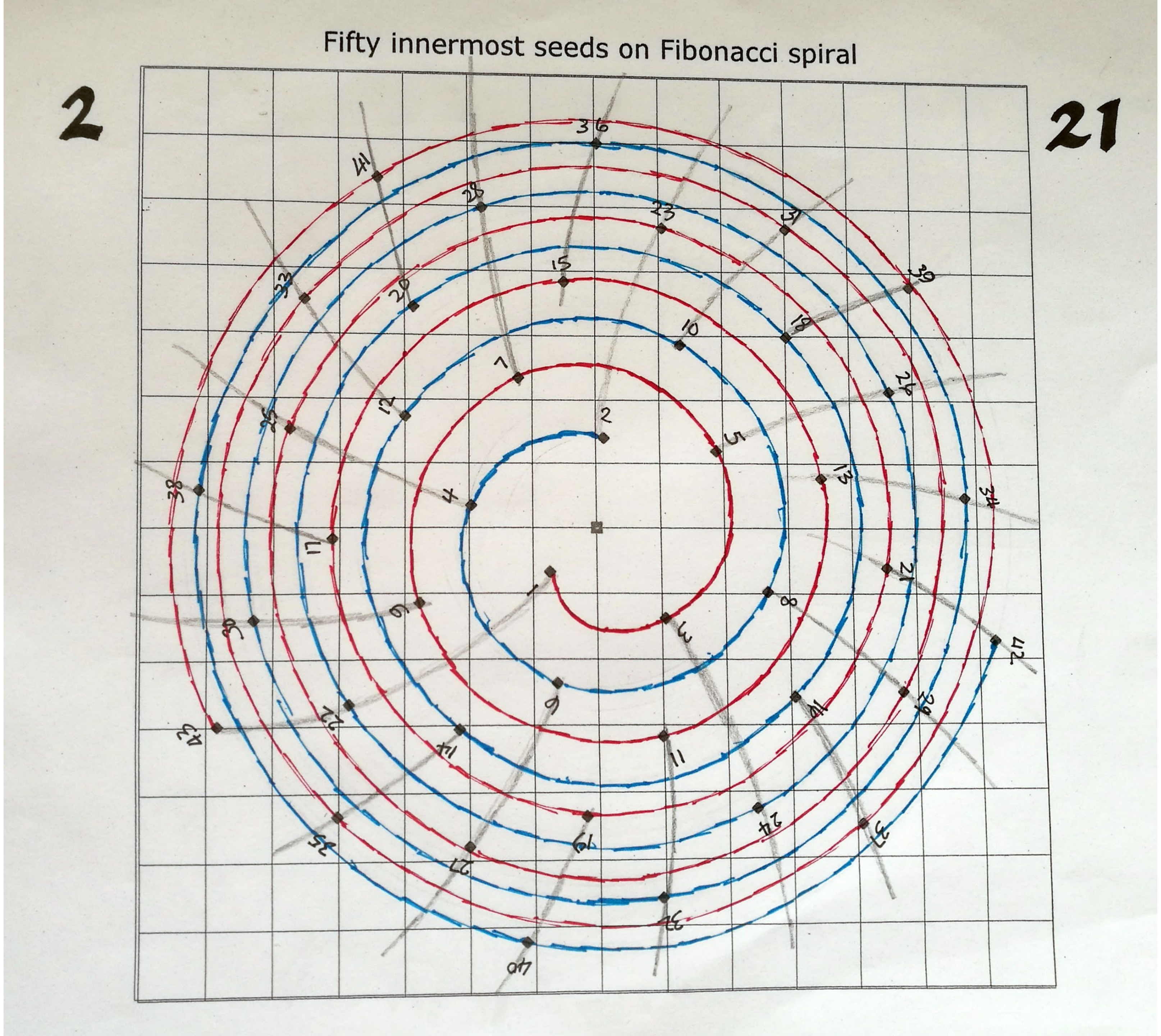


Two flat anticlockwise spirals (red and blue) connect every second seed.

The 21 steep, black, clockwise spirals connect every 21st seed in the sequence.

(The Fibonacci series goes: 1 **2** 3 5 8 13 **21** 34)

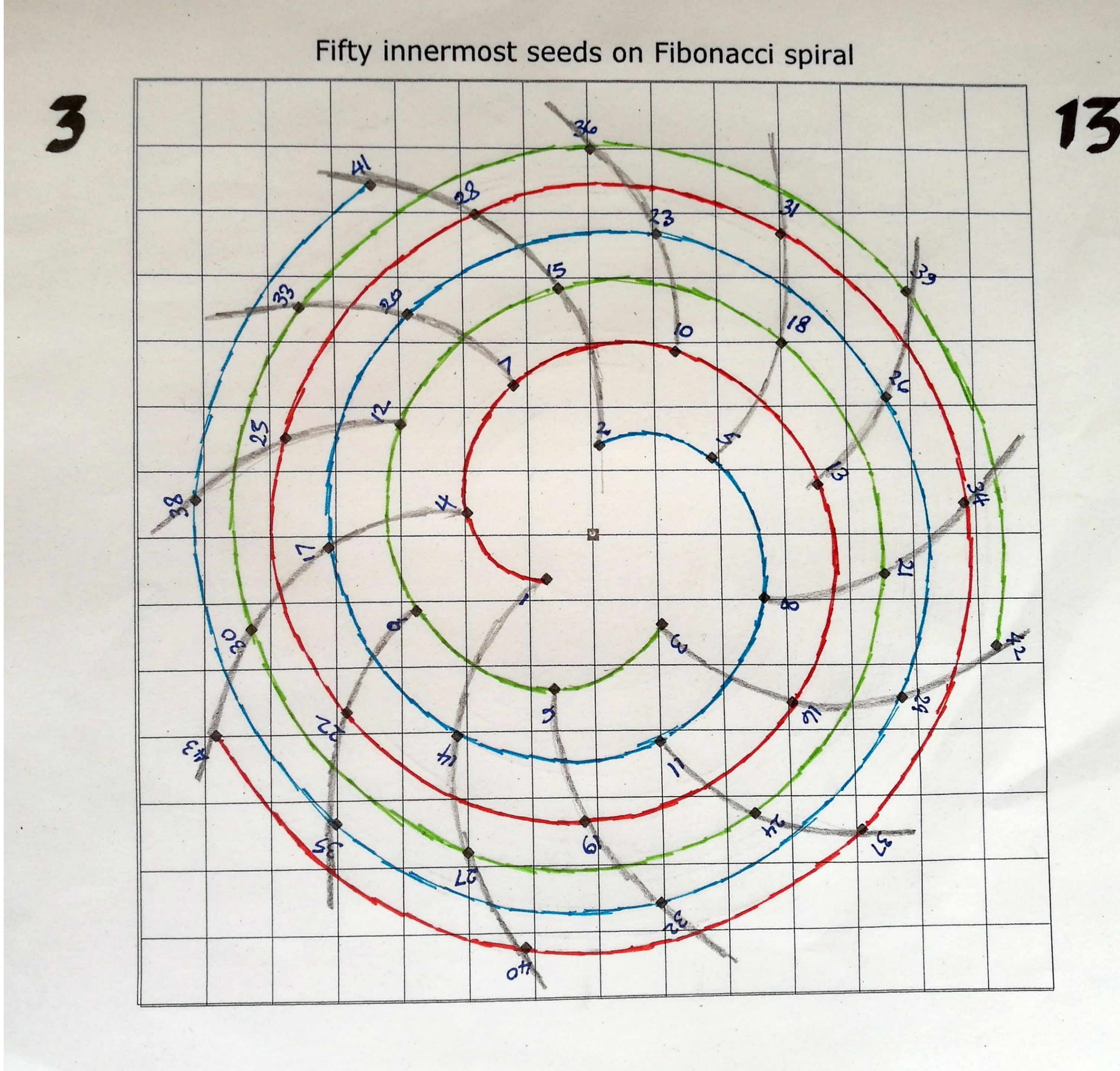
◦



Three flat spirals connect every third seed.。

13 steep spirals connect every 13th seed in the sequence.

(The Fibonacci series goes: 1 2 **3** 5 8 **13** 21 34)

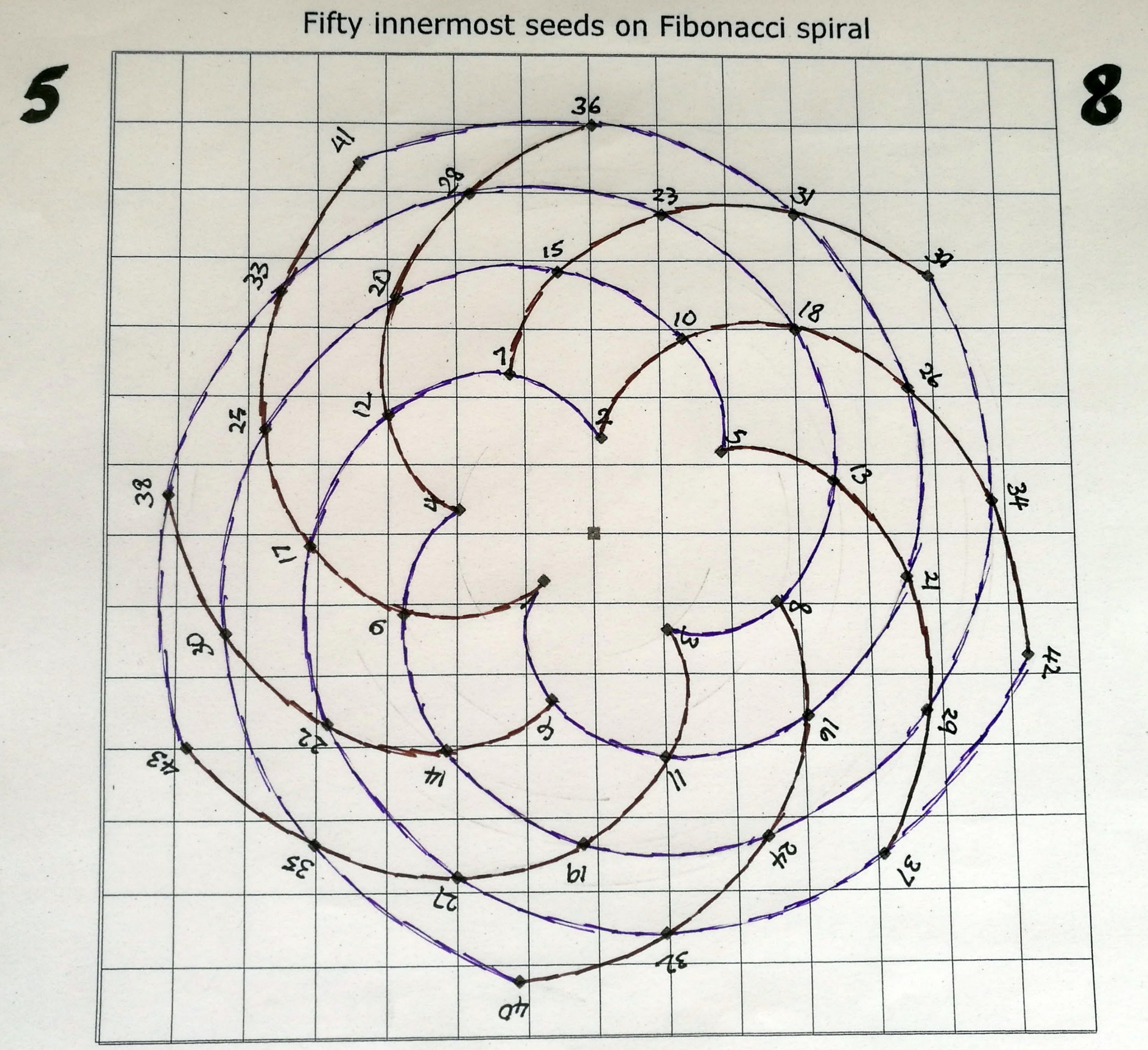


Five flatter spirals connect every fifth seed.

Eight steeper spirals connect every 8^{th} seed in the sequence.

(The Fibonacci series goes:

1 2 3 **5** **8** 13 21 34)



Note how each seed has about the same amount of space around it as every other seed.