Pythagorean Geometry

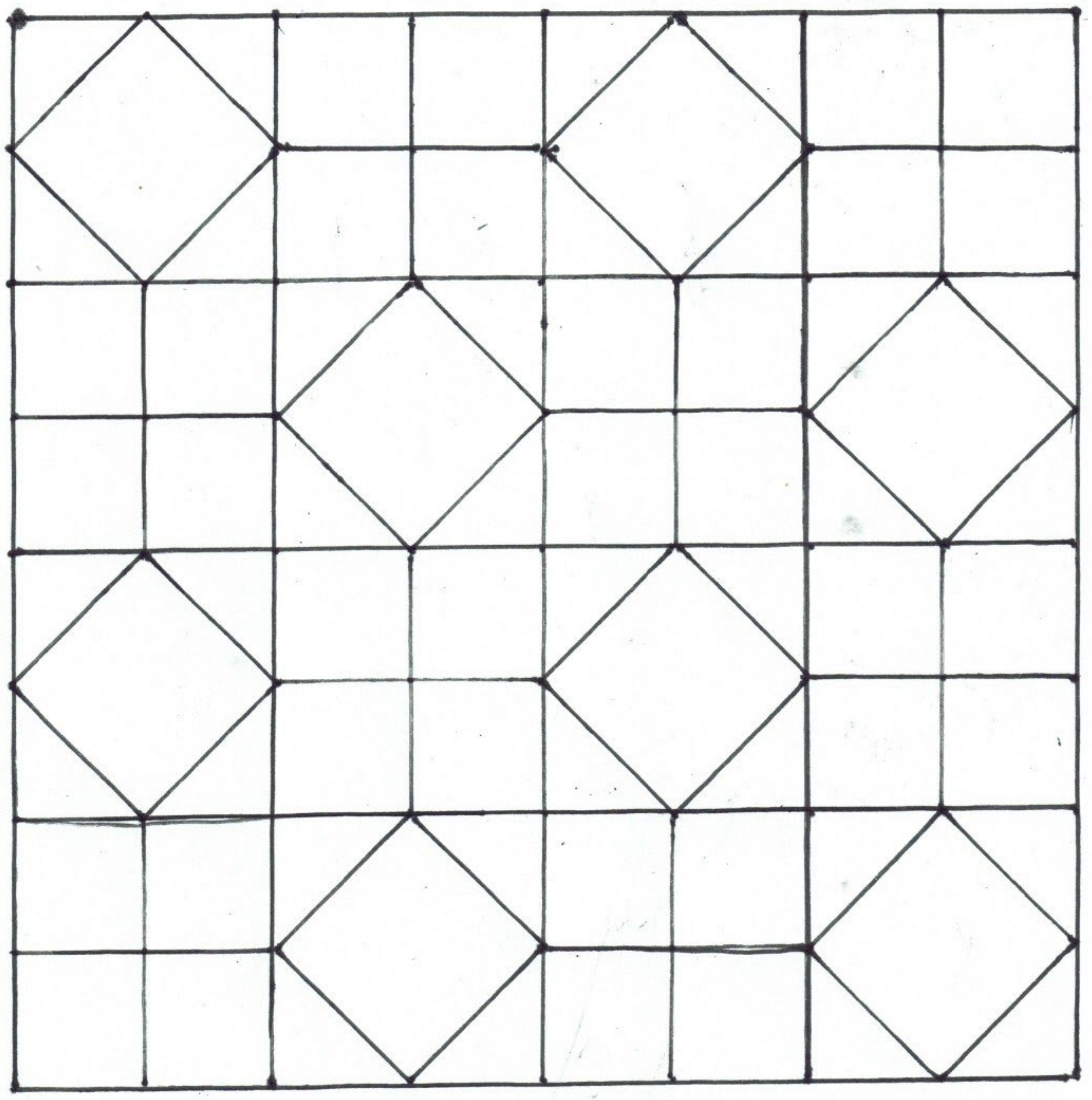
Representing Space and Time

What might Pythagoras have seen?

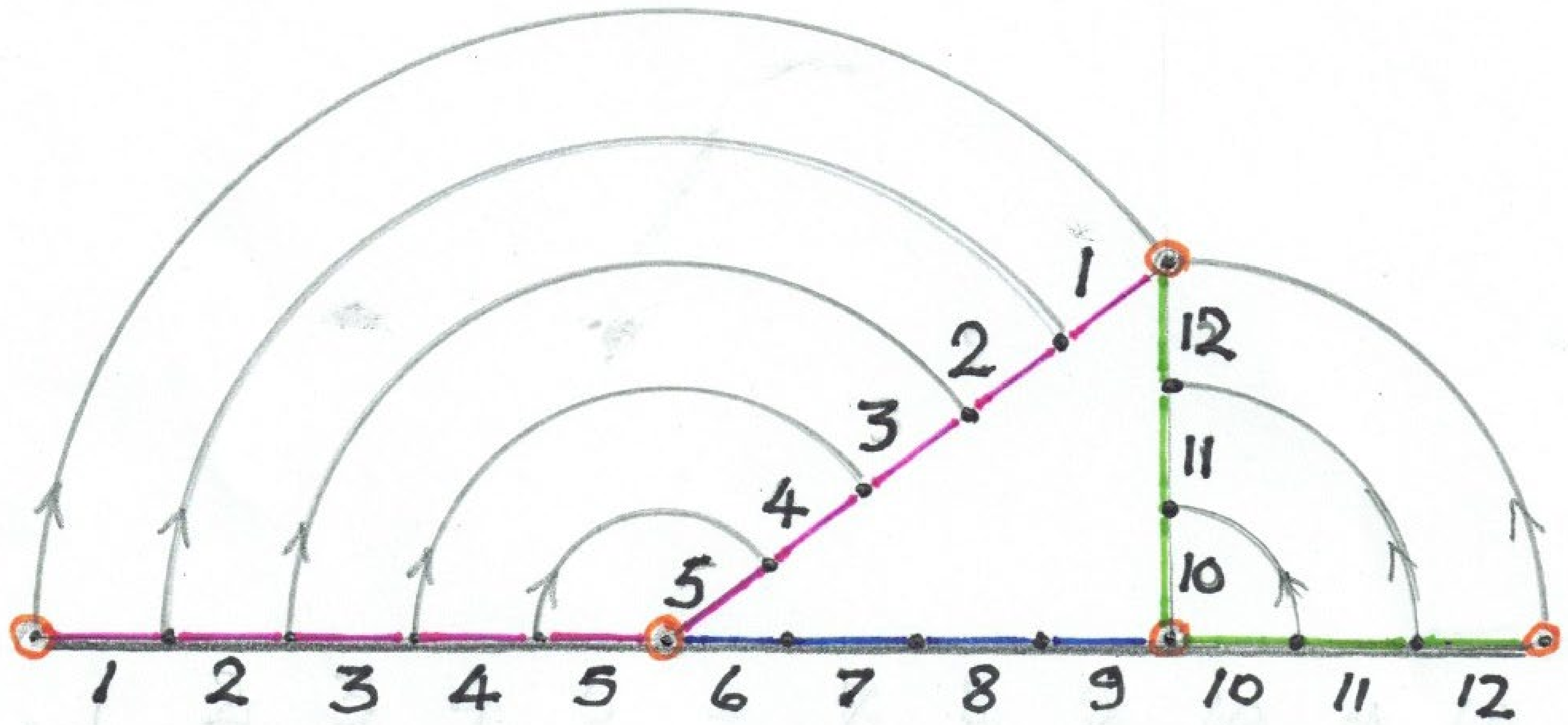
- What we know about Pythagoras was not written by him but by historians several centuries after he was alive, so we can only speculate on what stimulated him.
- The essence of the squares on a right triangle was known by the Babylonians many years before he lived, but he may have seen floor tile patterns in public buildings using squares and triangles. I seems that Pythagoras was the first to study them in detail.
- While he was in Egypt, he may have seen men, called “rope-stretchers” surveying the banks of the Nile after the spring floods to re-establish the boundaries of farmers’ land for tax purposes. They used a knotted rope, often 100 cubits long to measure the sides, but used a section 12 cubits long to create a right-angle [3-4-5] when they reached a corner.
- Pythagoras was also very interested in number patterns.

Floor tile pattern

Can you see that the larger square is twice the size of the smaller one, and the triangle is half the smaller square?

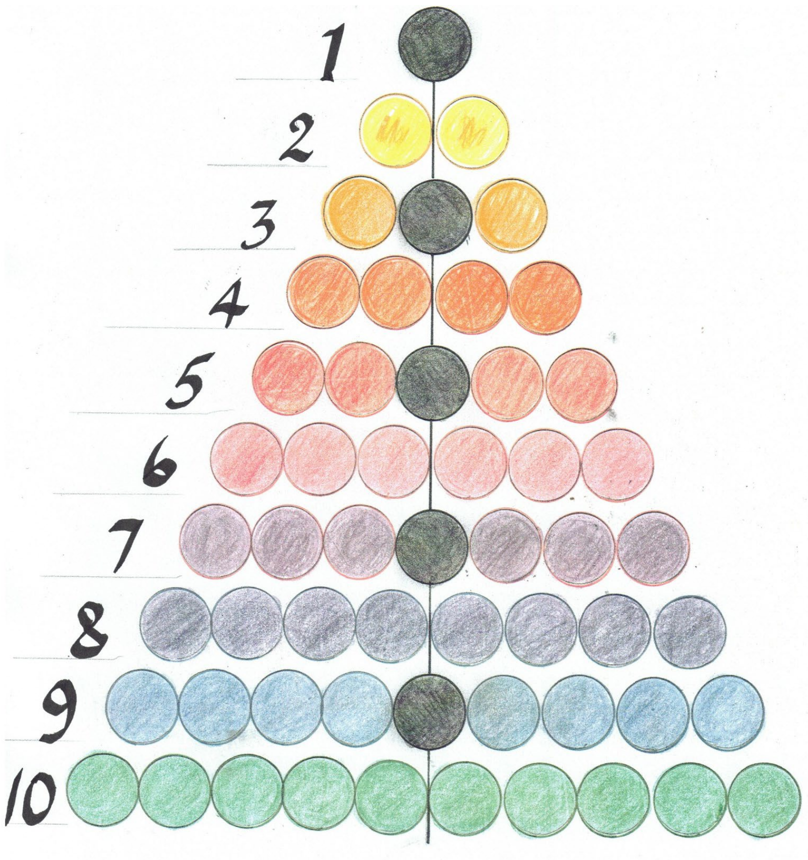


Surveyor's corner rope
for creating a right-angle



Odds and Evens

- Look at the numbers 1 to 10 stacked one above the other.
- What do you notice?
- There is a line of symmetry.
- Every second number has a dark centre disc.
- These are the ODD numbers
- The others are the EVEN numbers that can be split into two equal halves.
- Let us, like Pythagoras, consider separately the ODDS and the EVENS – relating to Space/geometry (odd) and Time/music. (even)
- (Why are men considered Odd and women Even?)



Prime Factors

(1)

2

3

2x2

5

2x3

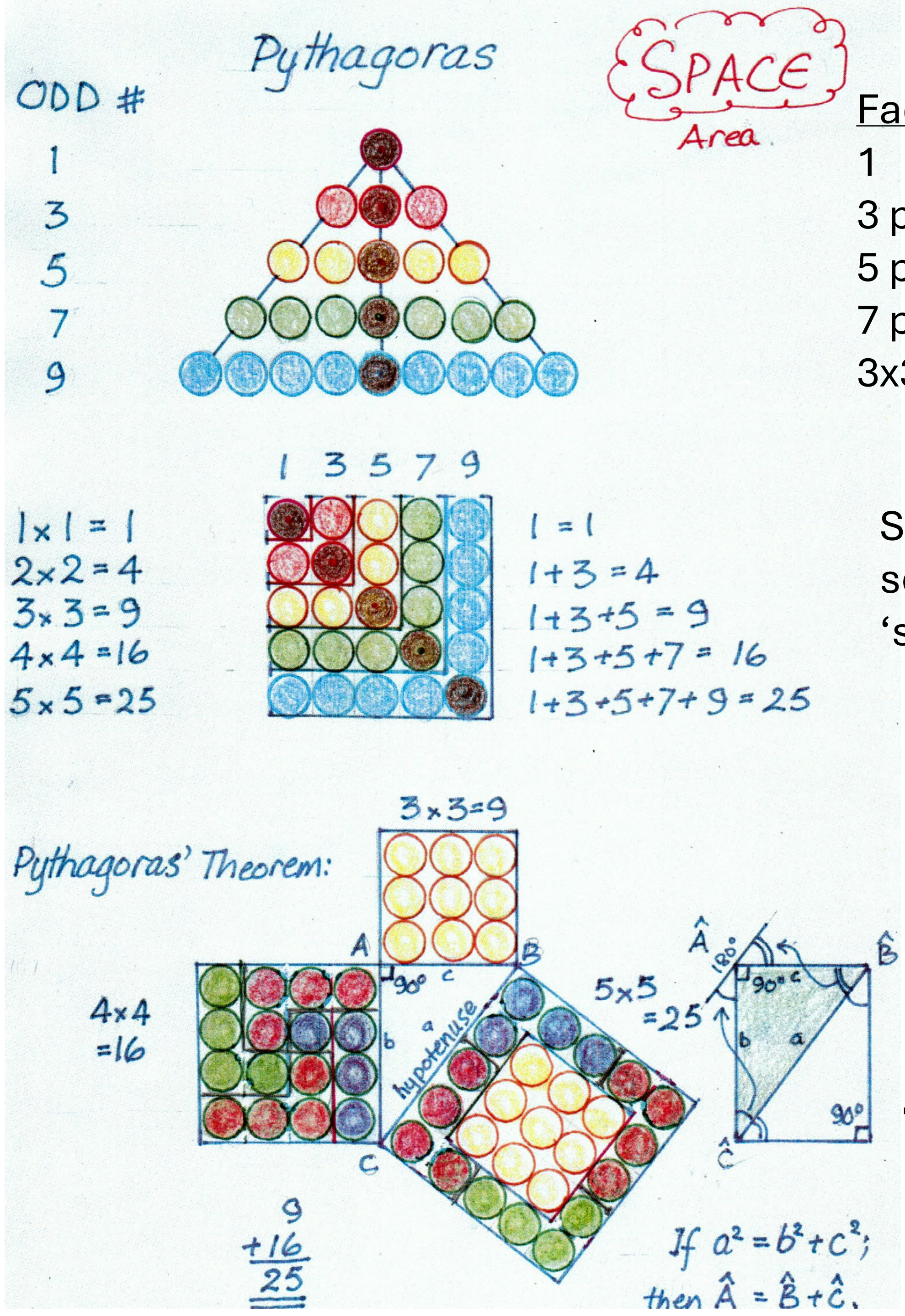
7

2x2x2

3x3

2x5

A **Prime Number** is a whole number greater than 1 that cannot be exactly divided by any whole number other than itself and 1 (e.g. 2, 3, 5, 7, 11).



Factors:

1
3 prime
5 prime
7 prime
3x3 square

Squares

Sum of Odd number sequences create 'squares'

This is the most famous example when the lengths of the sides are all whole numbers: **3-4-5.**
Some other 'triples' are 5-12-13; 7-24-25; 9-40-41 (and their multiples).

If a^2 **>=<** $b^2 + c^2$

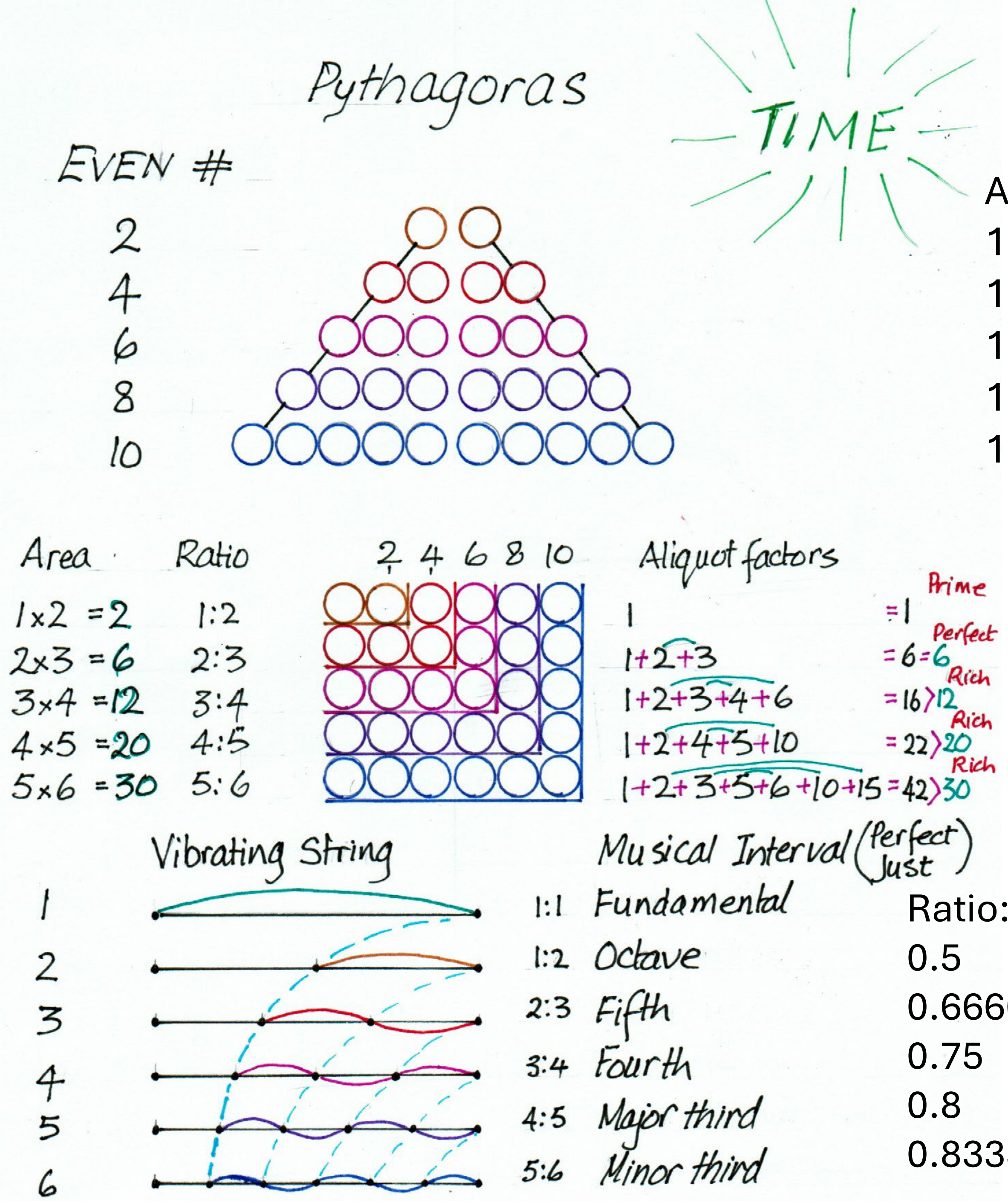
Then $\measuredangle A$ **>=<** $\measuredangle B + \measuredangle C$

(greater than, equal, less than)

A number is only classified as Rich or Abundant ifactors is greaf the sum of its Aliquot ter than the number itself. 12 is the first Rich number!
1+2+3+4+6 = 16 > 12

RECTANGLES

Basis for the Pythagorean harmonious musical scale. A taut string/wire vibrates in a whole number of parts. This scale was only replaced in 19th Century by the 12 semi-tone equal-tempered Octave (based on the 12th root of 2)

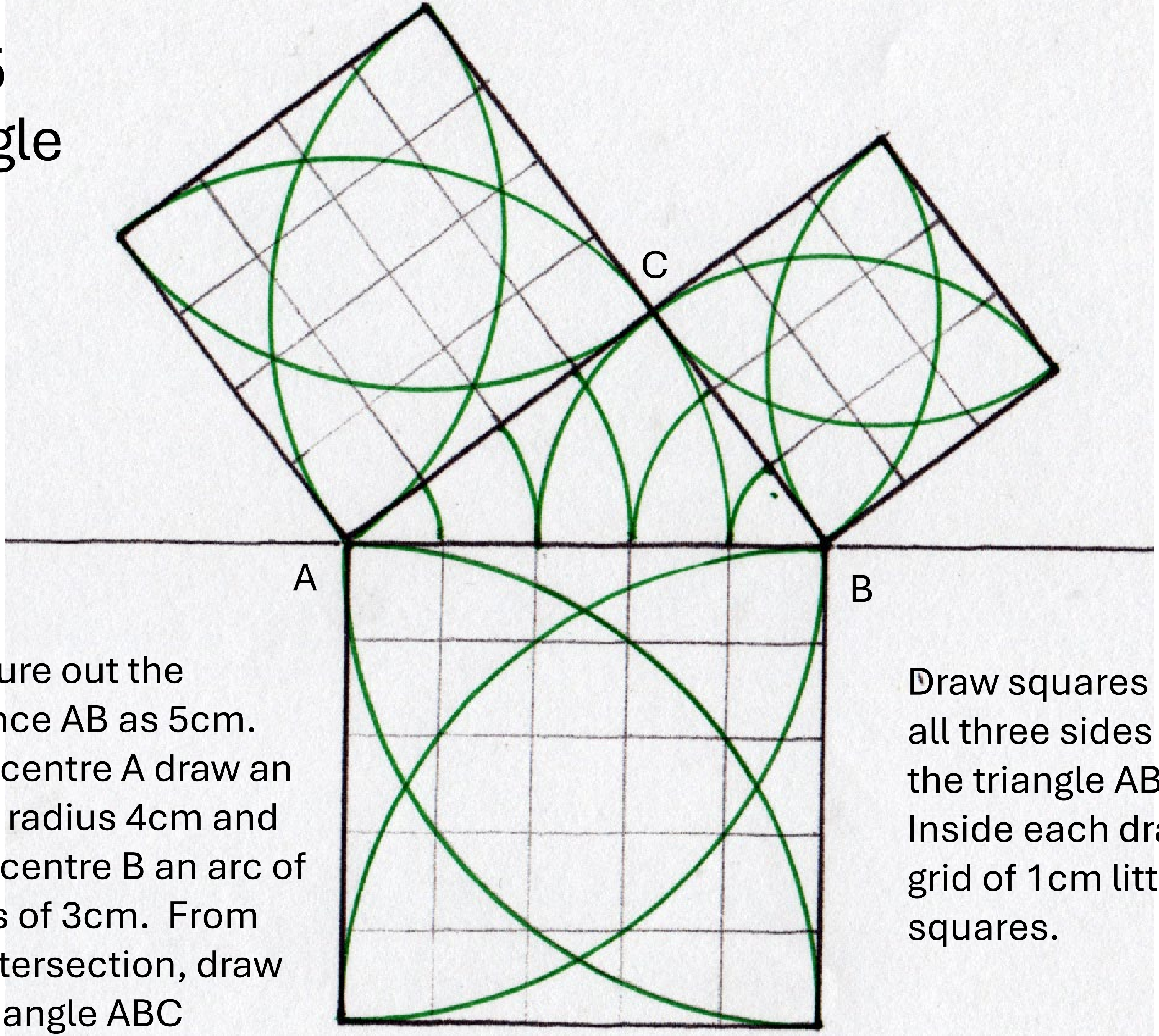


MUSIC

Aliquot Factors (all)
1 prime
1+2 < 4 poor
1+2+3 = 6 perfect
1+2+4 < 8 poor
1+2+5 < 10 poor

Ratio: 1
0.5
0.66666...
0.75
0.8
0.83333...

3-4-5 Triangle



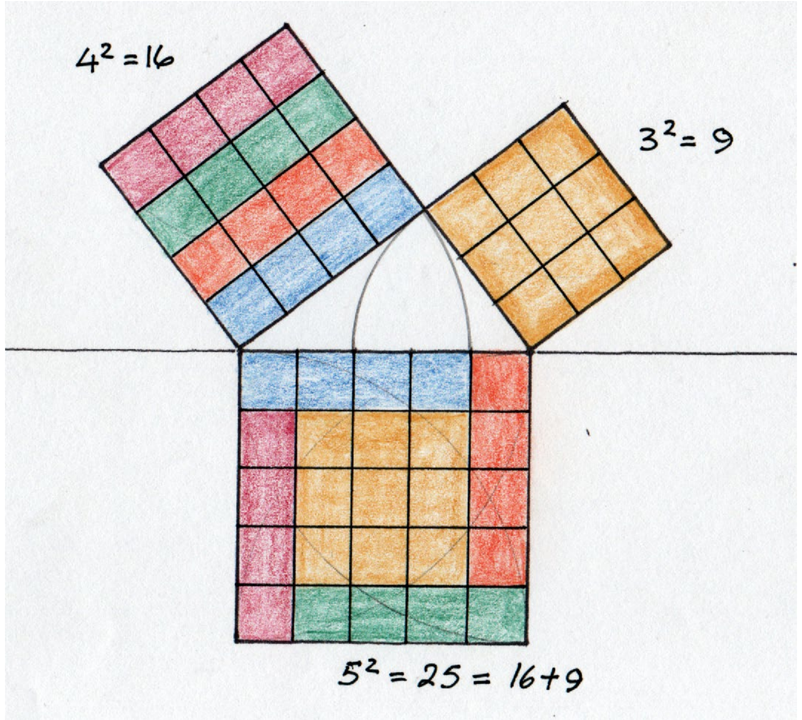
Measure out the distance AB as 5cm. From centre A draw an arc of radius 4cm and from centre B an arc of radius of 3cm. From the intersection, draw the triangle ABC

Draw squares on all three sides of the triangle ABC. Inside each draw a grid of 1cm little squares.

Pythagoras's Theorem

- Pythagoras' Theorem is most often stated as:
  "The square on the hypotenuse (longest side) of a right-angled triangle is equal to the sum of the squares on the other two sides."

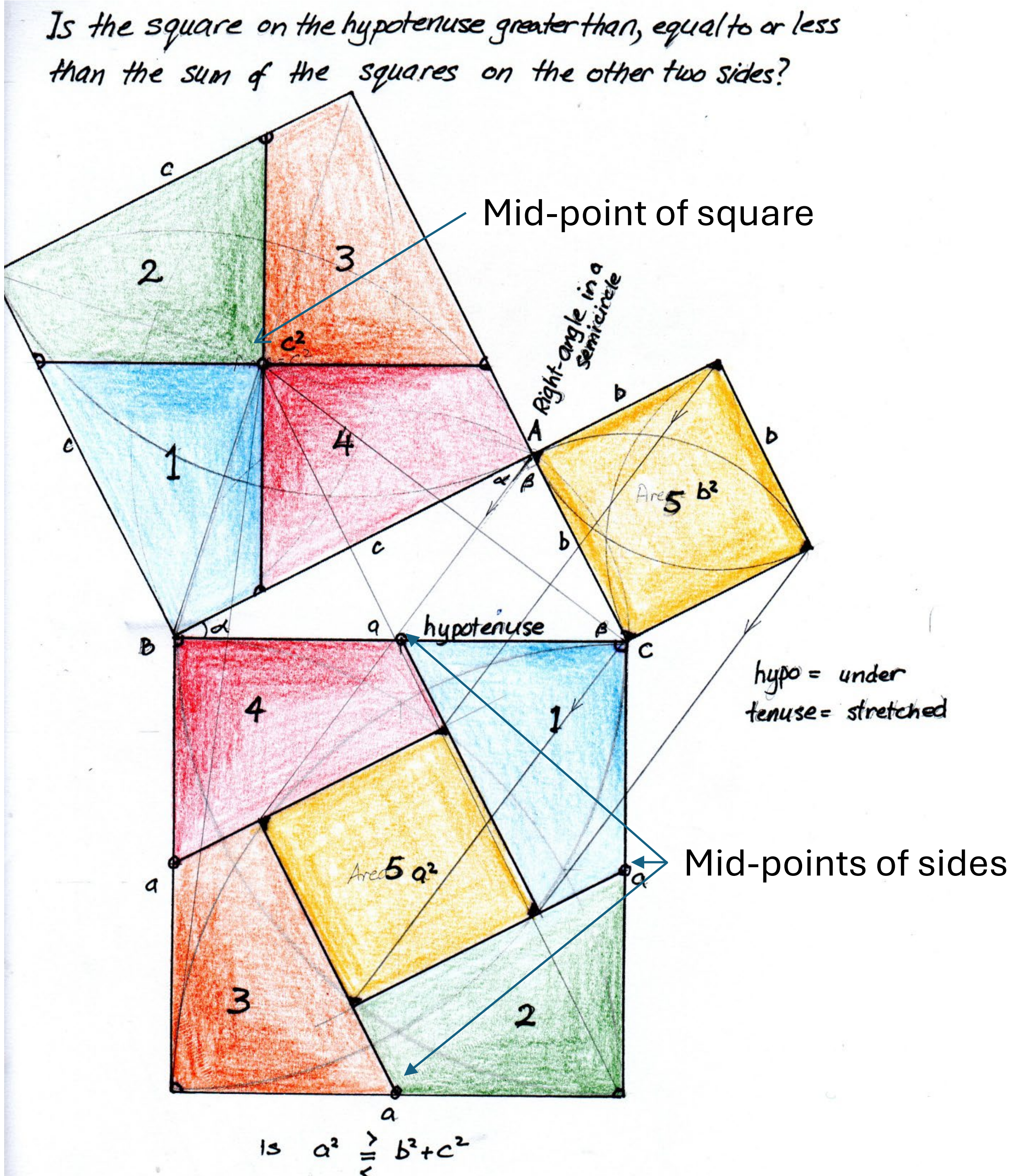
Every unit square in the two upper squares has a position in the lower square.
(note the corresponding colours)



Pythagoras Theorem

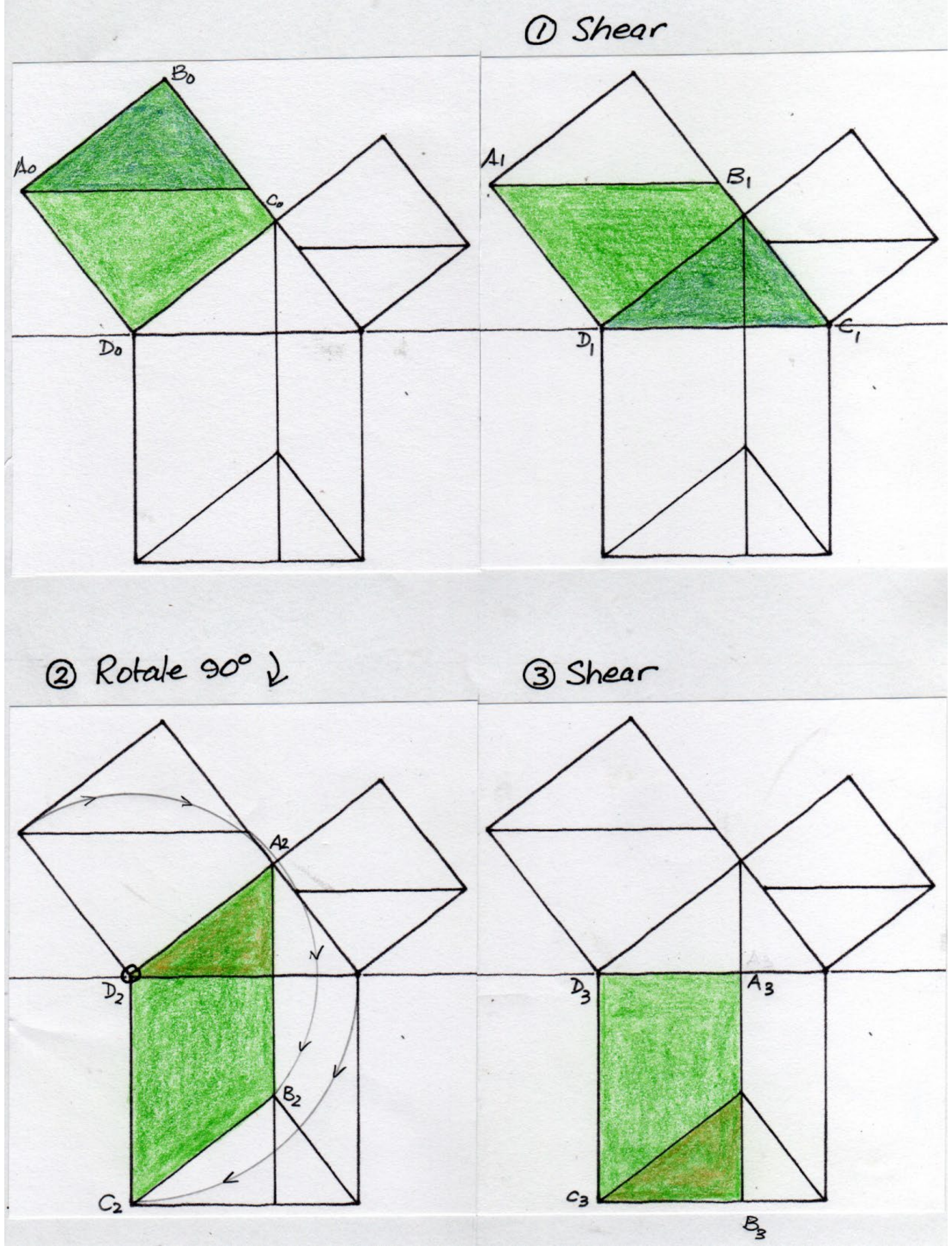
In the following diagrams the lengths of the sides can be any length; that is they may not be whole numbers of units.

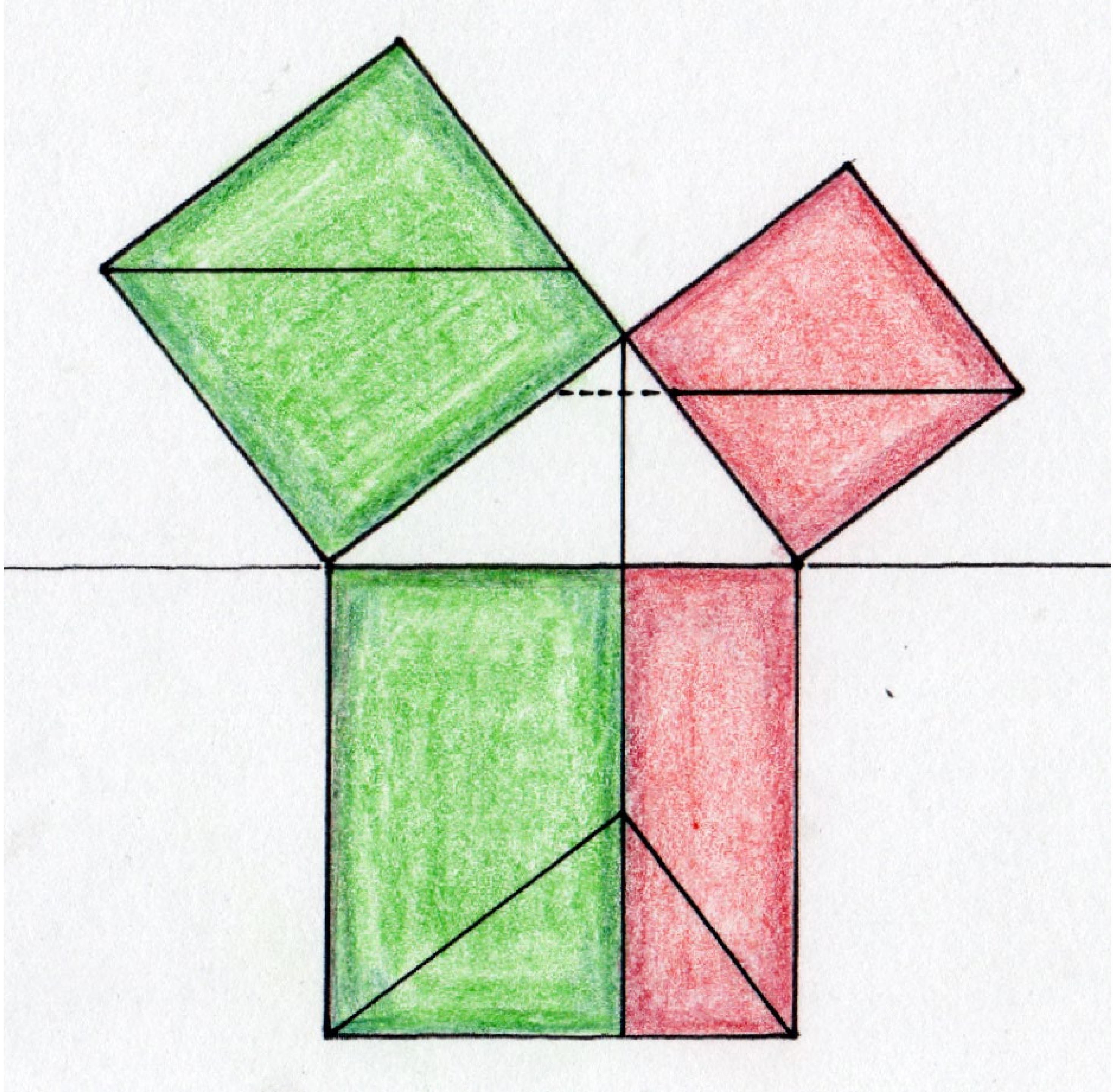
Can you prove to yourself that the divisions of the squares are the same size in each position?
Compare angles (which are right angles?) and compare the length of the sides.
The transformation process is called '**translation**' – where each of the 5 element areas move to a place in the largest triangle without rotating or turning or changing shape.

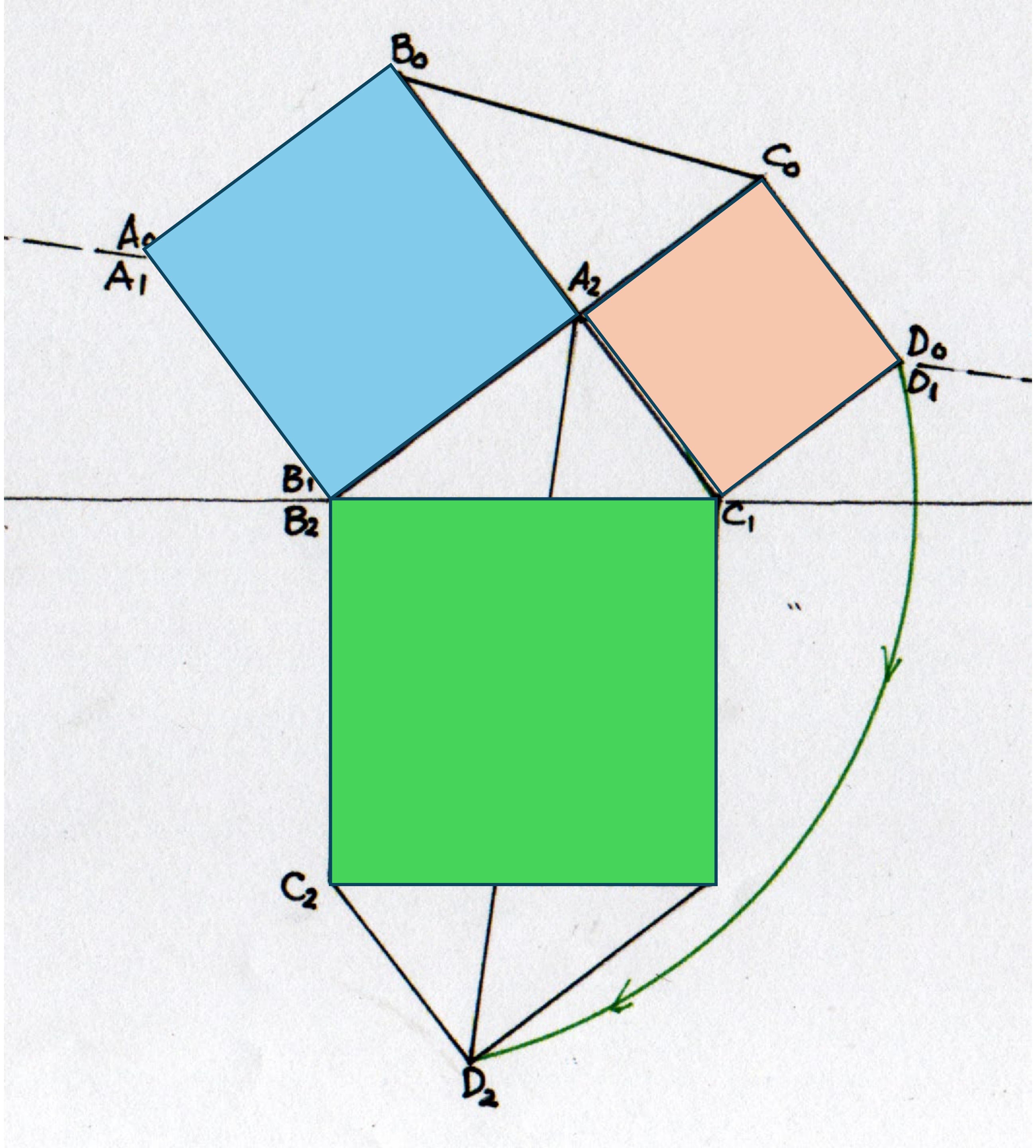


The transformation of **square** into a **rectangle** by SHEAR and ROTATION

Here we are concerned with AREA, not the length of the sides.
The lower or largest square is divided into two rectangles by a line through the apex or right-angle of the TRIANGLE

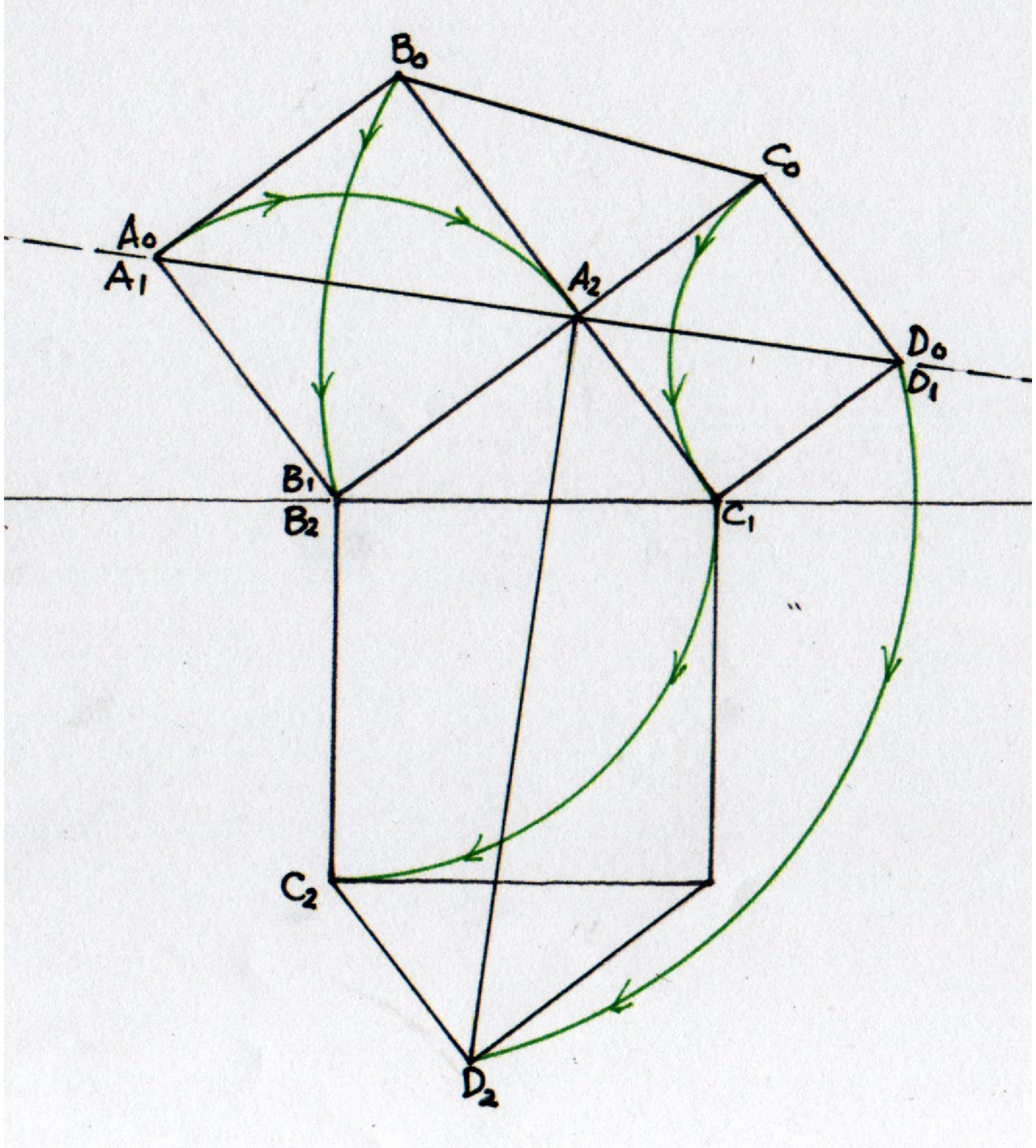


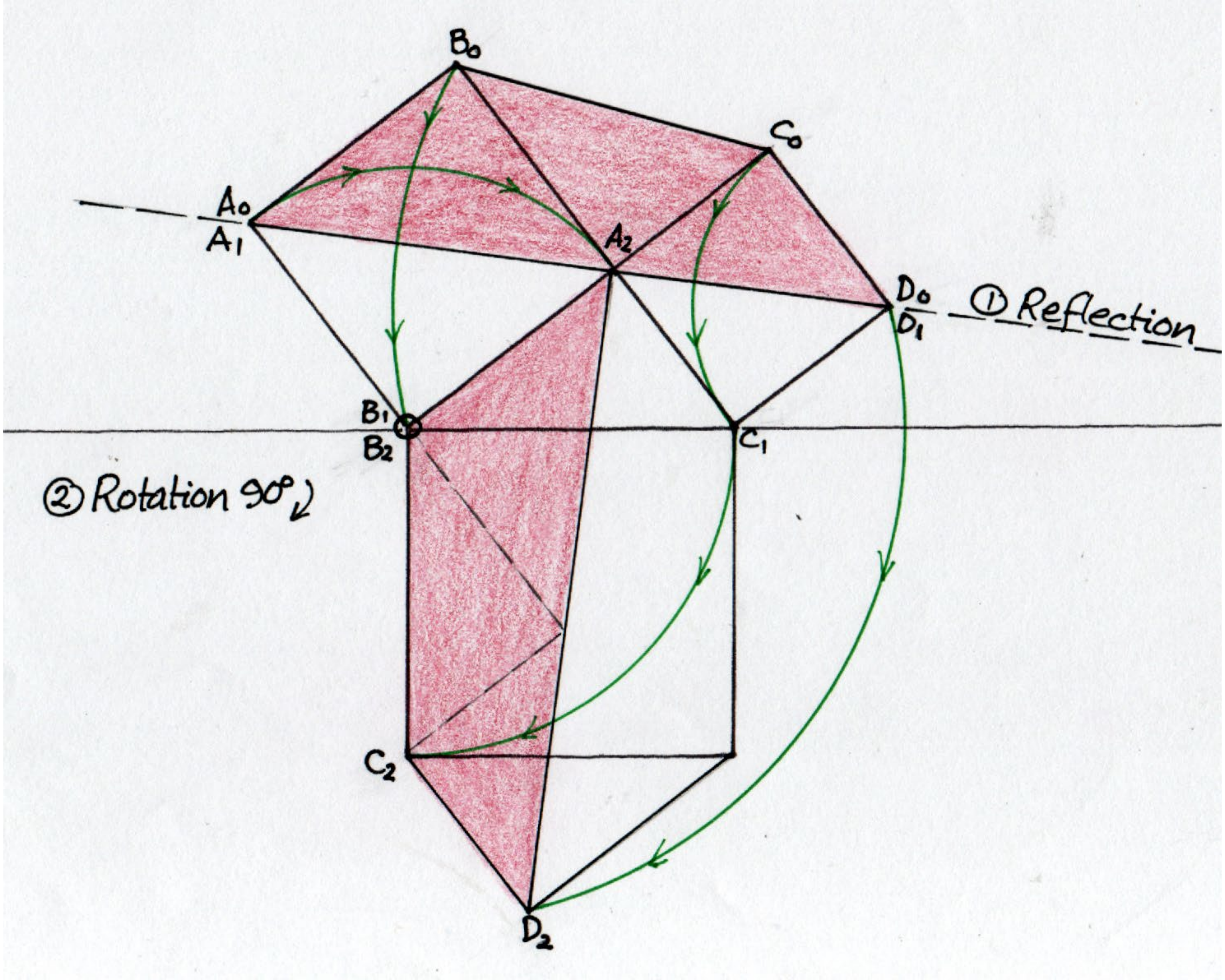




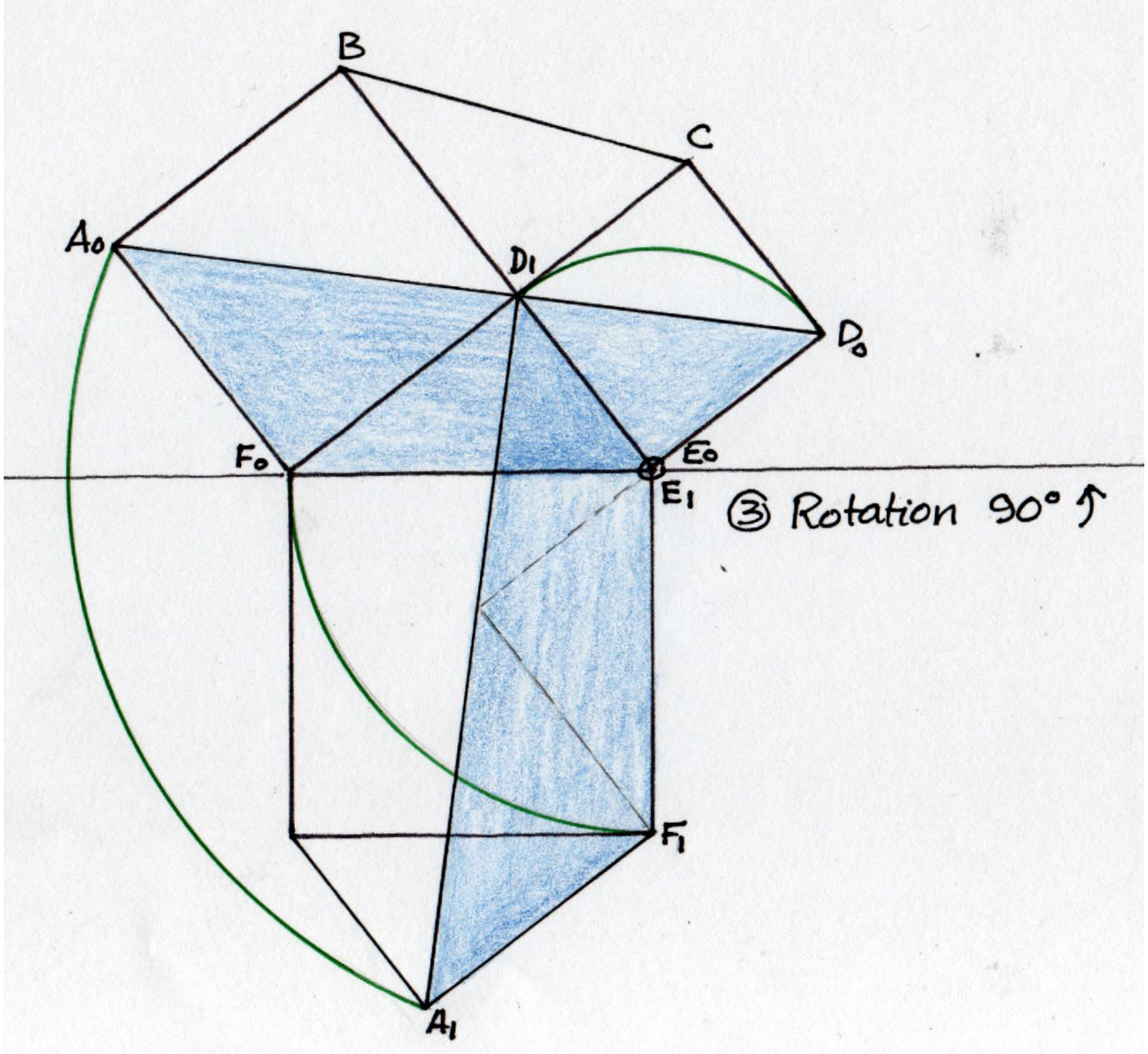
The transformation of 2 squares into the large square by REFLECTION and ROTATION

Transformation using: **Reflection** and **Rotation**

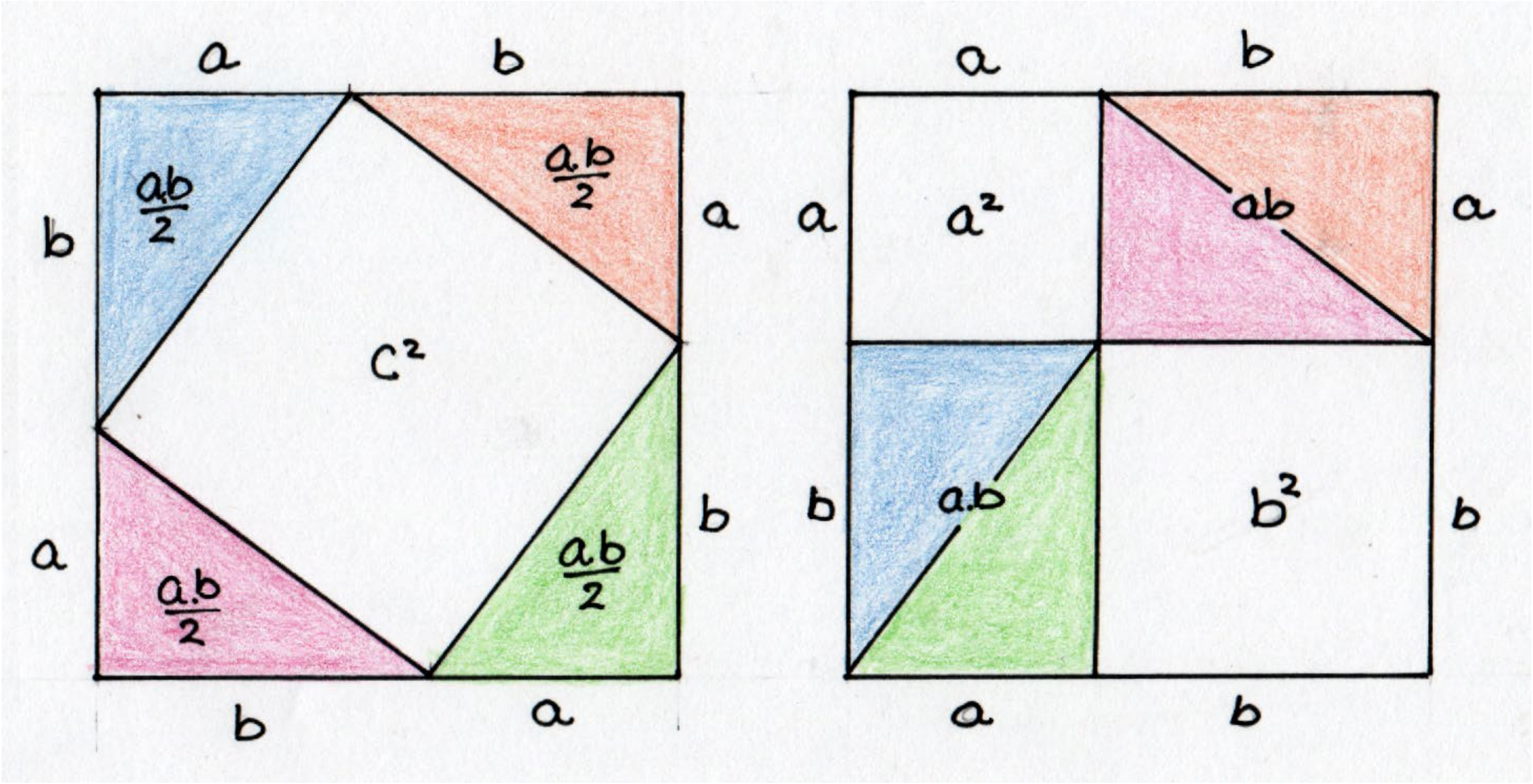




B₀
C₀
A₀
A₁
A₂
D₀
D₁
① Reflection
B₁
B₂
C₁
② Rotation 90°
C₂
D₂


B
C
A0
D1
D0
F0
E0
E1
③ Rotation 90° ↑
F1
A1

An algebraic proof of Pythagoras' Theorem
$a^2 + b^2 = c^2$



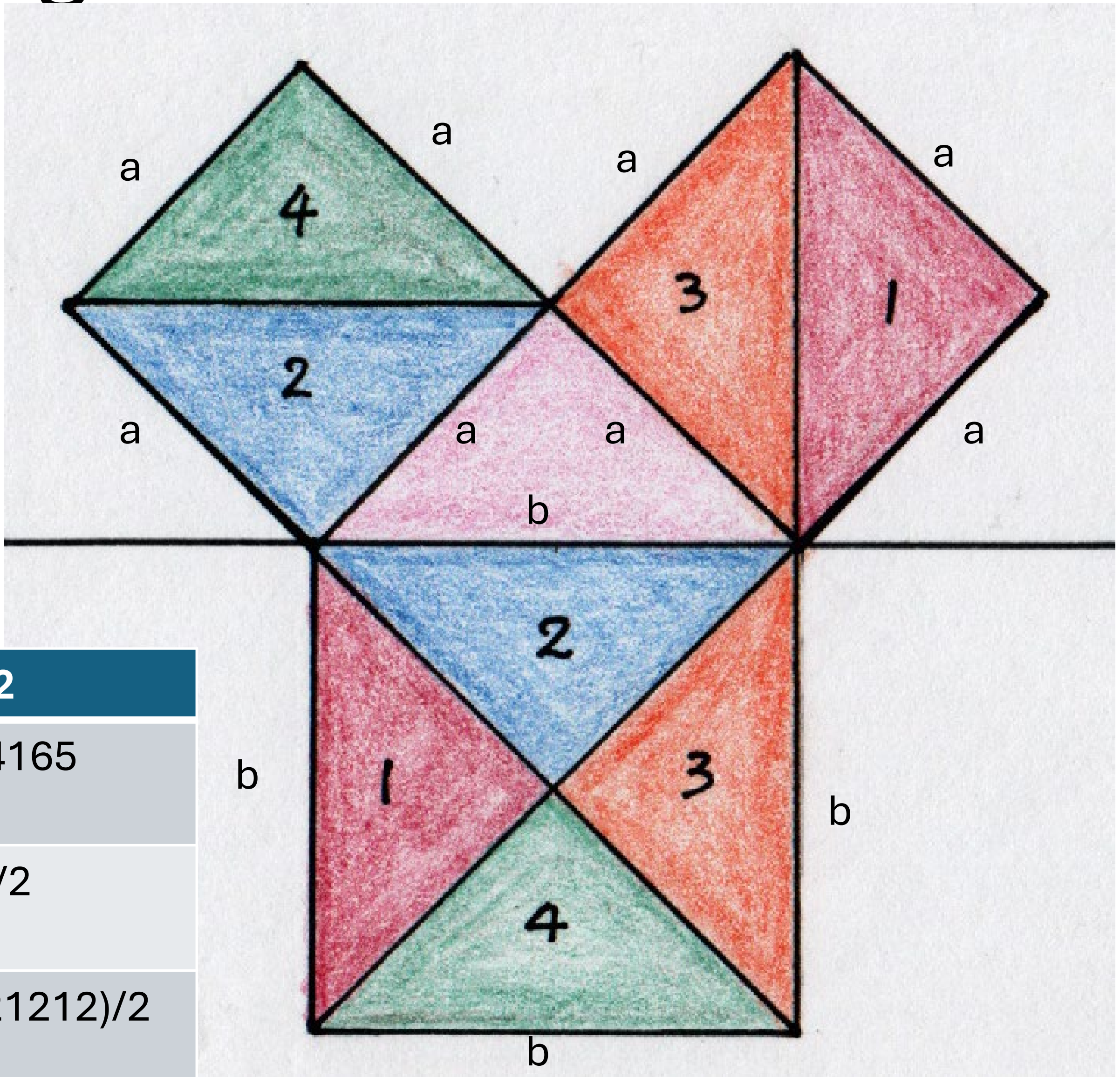
- It is fascinating how many ways there are of demonstrating or proving the truth of Pythagoras' Theorem.
- I have shown just a few of the 370 proofs that have been discovered, algebraic and visual. Rudolf Steiner was keen that this theorem was part of the Waldorf School Curriculum.
- In the High School, students encounter theodolite surveying and Trigonometry.
- In Trigonometry, we find Pythagoras' Theorem modified and adapted to all triangles – not just right-angled triangles - using the Cosine Ratio. (=adjacent/hypotenuse)
- **$a^2 = b^2 + c^2 - 2.b.c.Cos(\measuredangle A)$**

Square Roots

- In Pythagoras' time, IRRATIONAL numbers had not been explored; they were "impossible" or perhaps even 'ineffable'.
- Even decimals were not available.
- However, as soon as we look at triangles beyond the few 'triples' such as 3-4-5 we encounter square roots that are irrational. That cannot be expressed as a ratio or a repeating decimal., such as 1/3 = 0.3333333…
- The first one we encounter is $\sqrt{2}$. The next slide shows a process whereby such a number can be computed as an [endless] decimal.

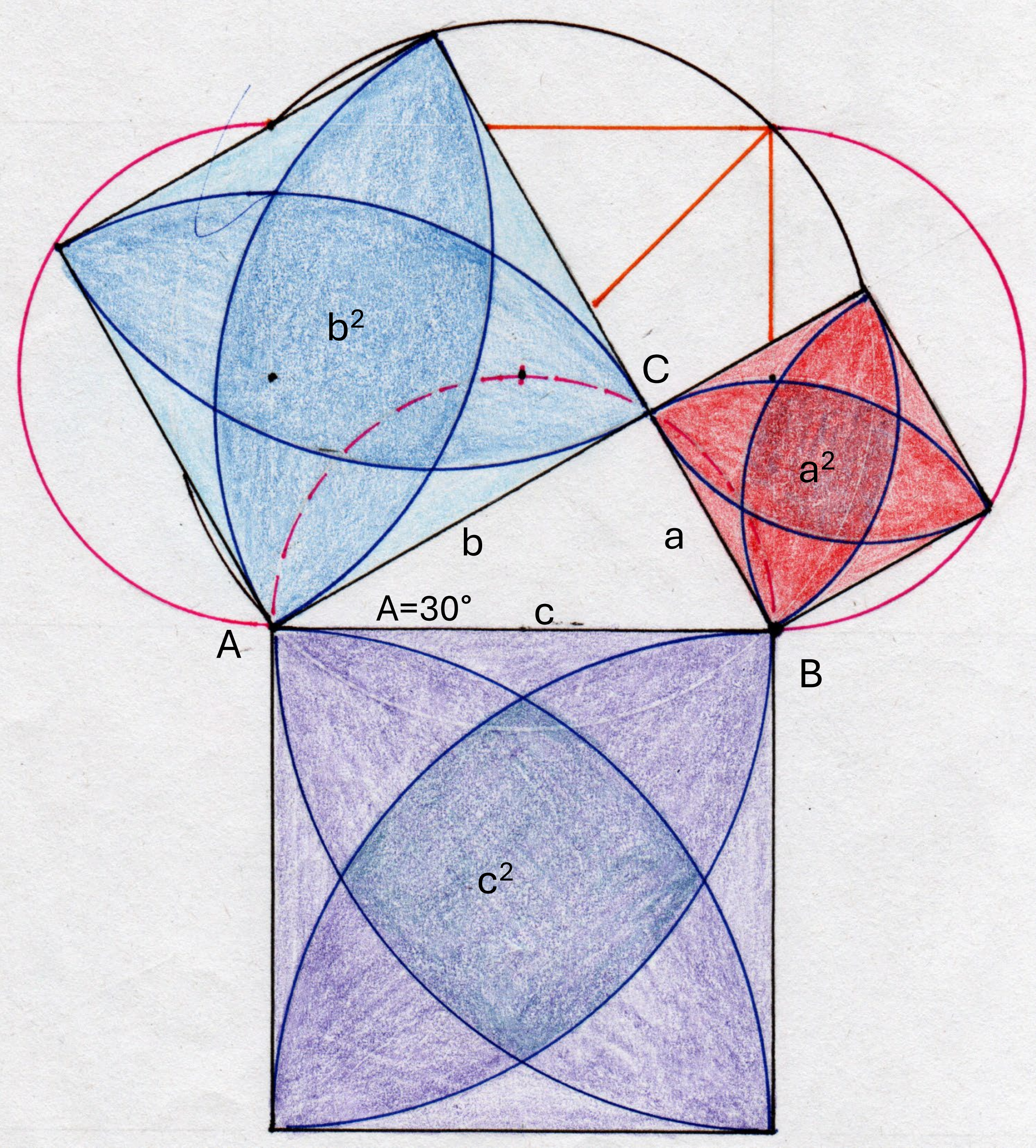
The Isosceles Right Triangle

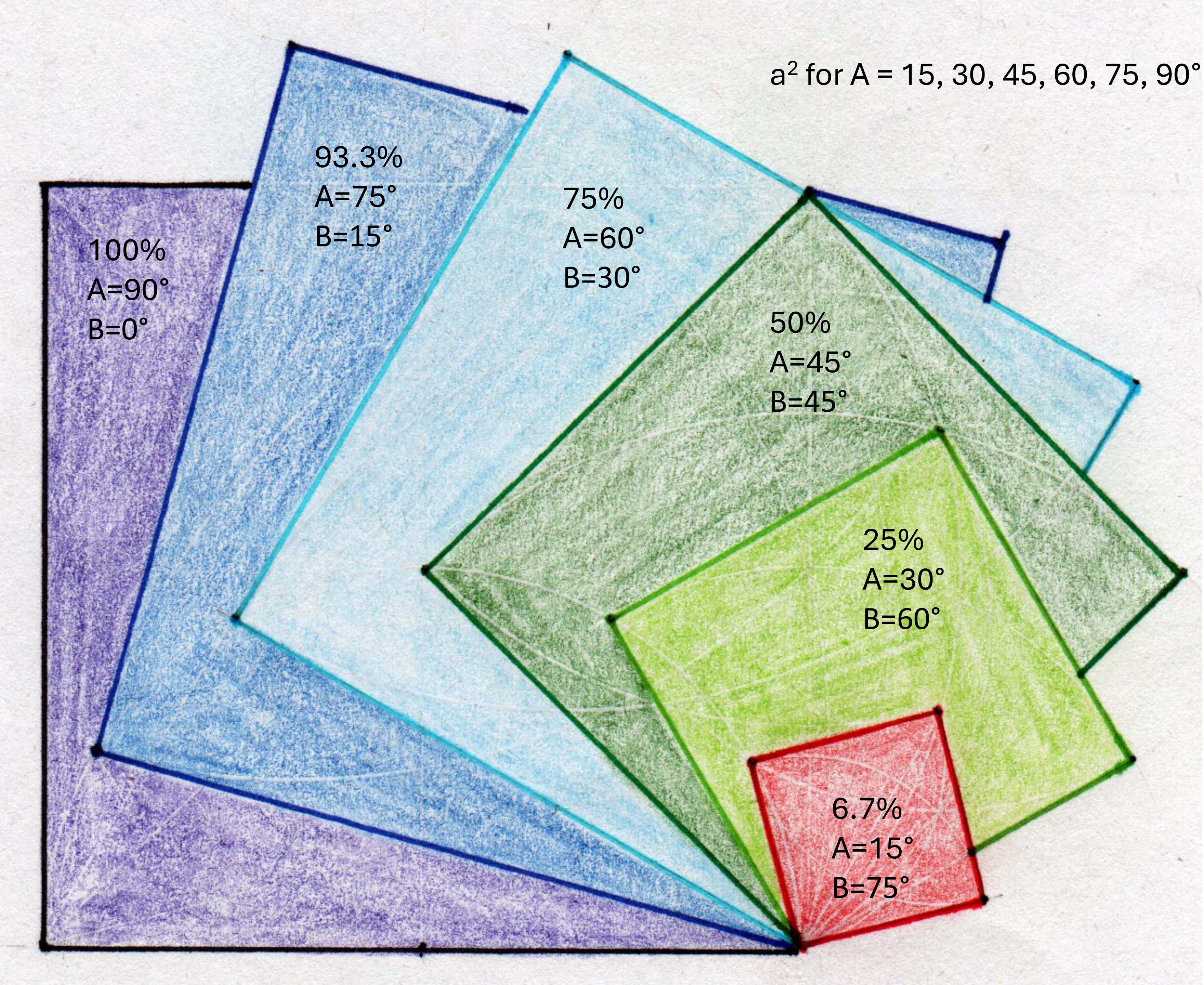
- The two small slides are of equal size, a^2; the larger square is twice the area: $b^2= 2*a^2$
- Thus $(b/a)^2= 2$, then using square roots, then b/a = √2. what ratio works?
- Imagine that if two very similar numbers: c*d=2, then c and its conjugate, d=2/c, will be very close to √2. One will be a bit high, the other a bit low, so we take the average or Mean value and try again.
- Below we have used this modern iterative process to calculate a decimal value for √2 to 8 decimal places

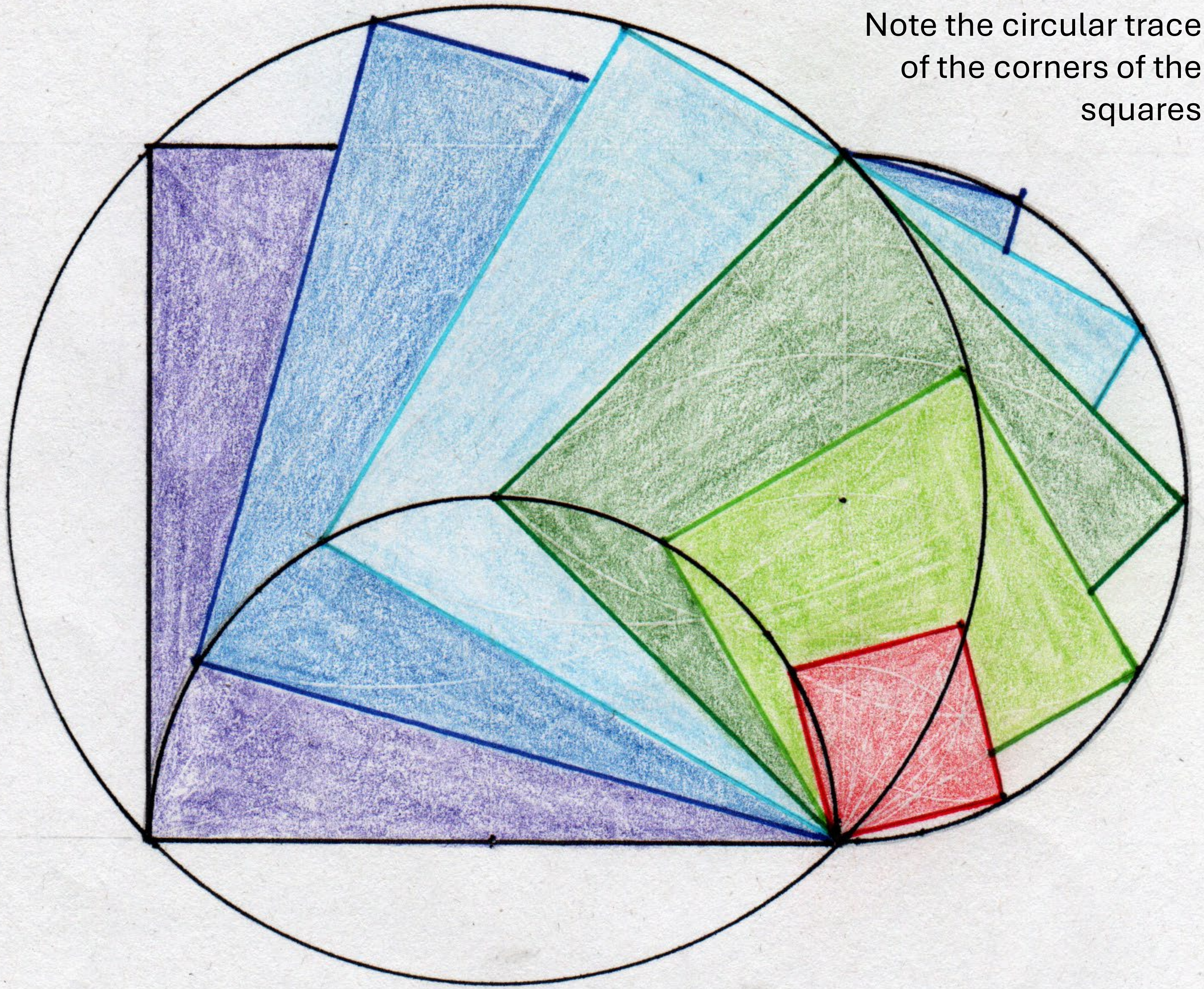


Rnd	Value of c	Conjugate d = 2/c	Mean (c+d)/2 = √2
#1	1.5 (a guess)	2/1.5 = 1.333	(1.5+1.333)/2= 1.4165
#2	1.4165	2/1.4165= 1.41193	(1.4165+1.41193)/2 = 1.414215
#3	1.414215	2/1.414215 = = 1,41421212	(1.414215+1.41421212)/2 **=1.41421356**

$a^2 + b^2 = c^2$




a^2 for A = 15, 30, 45, 60, 75, 90°
100%
A=90°
B=0°
93.3%
A=75°
B=15°
75%
A=60°
B=30°
50%
A=45°
B=45°
25%
A=30°
B=60°
6.7%
A=15°
B=75°



Note the circular trace of the corners of the squares

The End