Symmetry of Quadrilaterals

by Robin Bacchus

What is a quadrilateral?

- A quadrilateral is a plane polygon with 4 sides and 4 angles.. It may have no symmetry or many symmetrical aspects, such as:

a) Two or more sides of equal length

b) Two or more angles of equal size;

c) Opposite sides may be parallel.

d) There may be rotational symmetry.
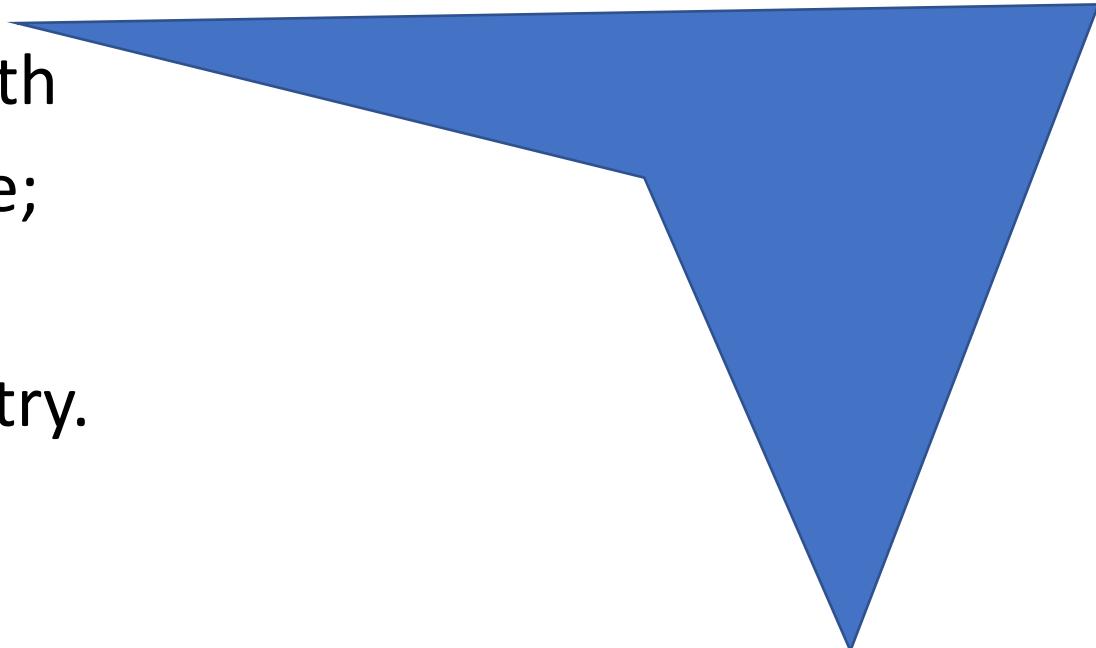
Kite

- 2 pairs of adjacent sides equal
  AB = AD
  CB = CD

- 1 pair of opposite angles equal.
  B = D

- No sides parallel.

- 1 diagonal bisects its angles.
  AC bisects A & C

- 1 axis of symmetry: bisector of angles A & C

- **Incircle**

KITE

Symmetry
1 diagonal
0 point

Sides: $\overline{AB} = \overline{AD} \neq \overline{CB} = \overline{CD}$

Angles: $\hat{B} = \hat{D} \neq \hat{A} \neq \hat{C}$

Parallel: none
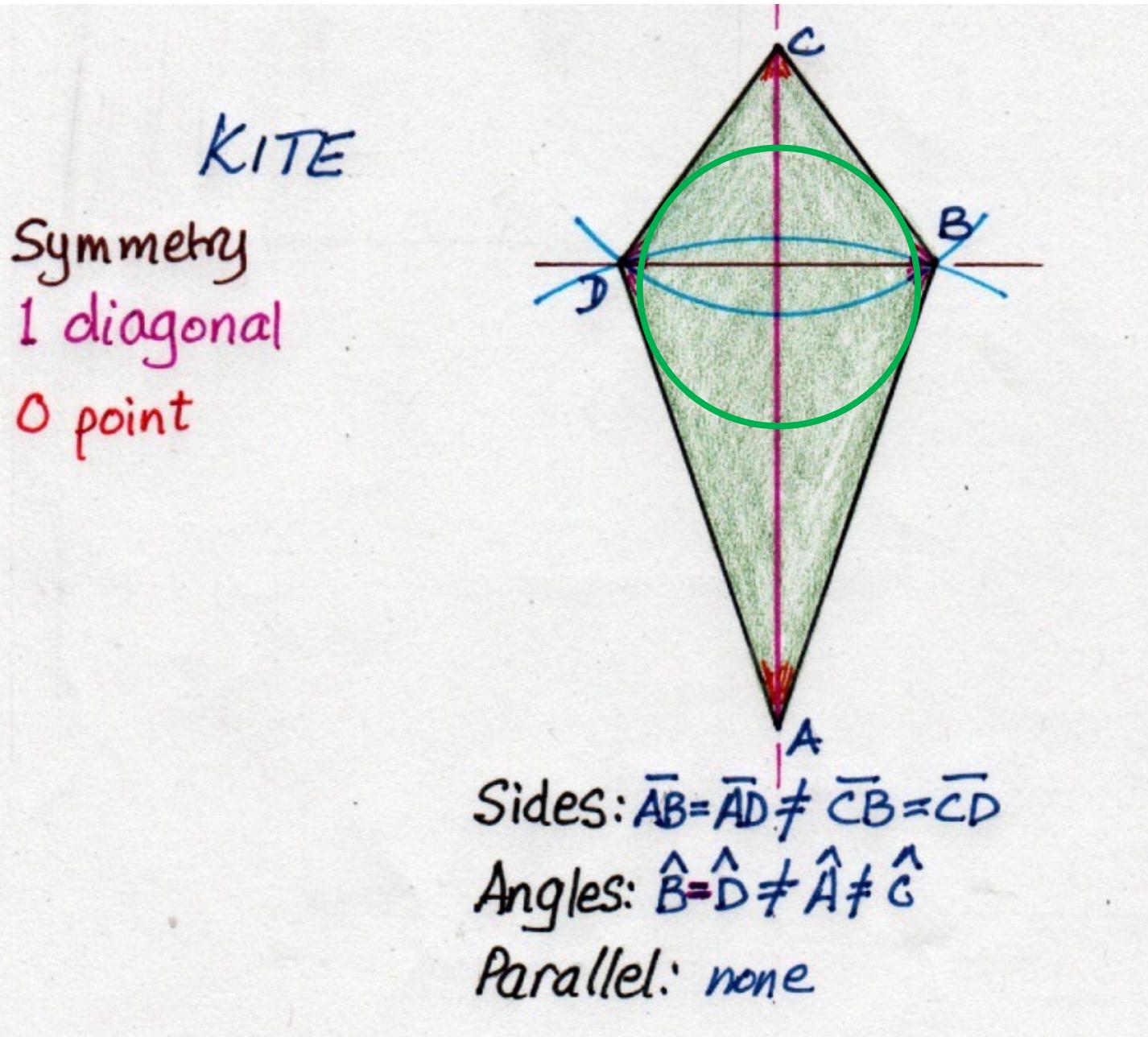
Isosceles Trapezium

- 1 pair of opposite sides equal.
  AD = BC

- 2 pairs of adjacent angles equ
  A = B; C = D

- 1 pair of opposite sides paralle
  AB || CD

- 1 axis of symmetry:
  bisector of CD and AB

- **Circumcircle**

TRAPEZIUM
Symmetry
⊥ bisector

Sides: $\overline{AD} = \overline{BC}$
Angles: $\hat{A} = \hat{B} \neq \hat{C} = \hat{D}$; $\hat{B} + \hat{C} = 180°$ (supplementary)
Parallel: $\overline{AB} \parallel \overline{DC}$
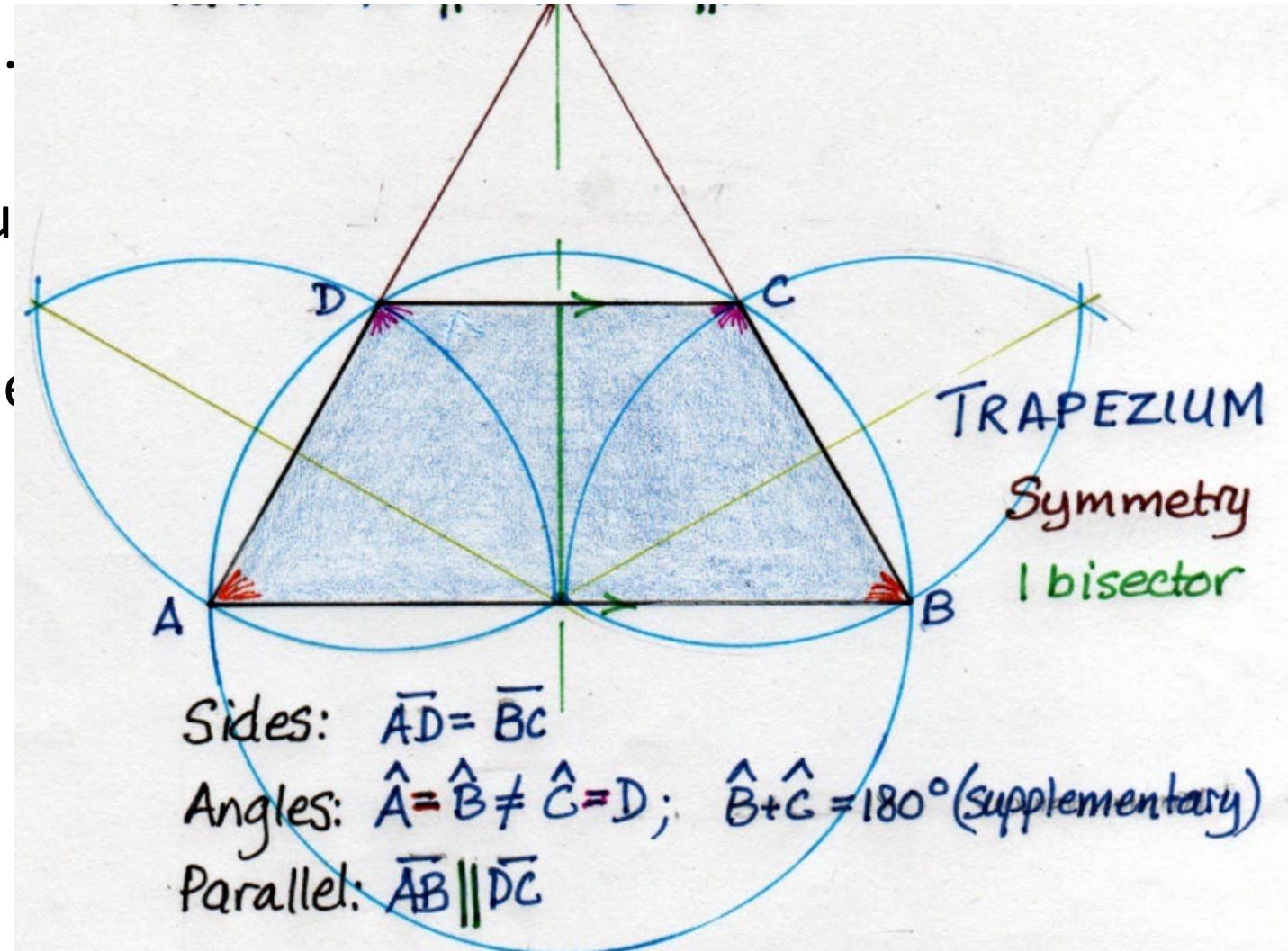
Rhombus

- Equilateral – all sides equal.
- 2 pairs of opposite angles equal A = C; B = D.
- 2 pairs of opposite sides parallel. AB || DC; BC || AD
- 2 axes of symmetry: both diagonals.
- 1 point of rotational symmetry (180°)
- **Incircle**

Equi-lateral

RHOMBUS

Symmetry
2 diagonals
1 point (180°)

Sides: $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$
Angles: $\hat{A} = \hat{C} \neq \hat{B} = \hat{D}$
Parallel: $\overline{AB} || \overline{DC}$; $\overline{BC} || \overline{AD}$
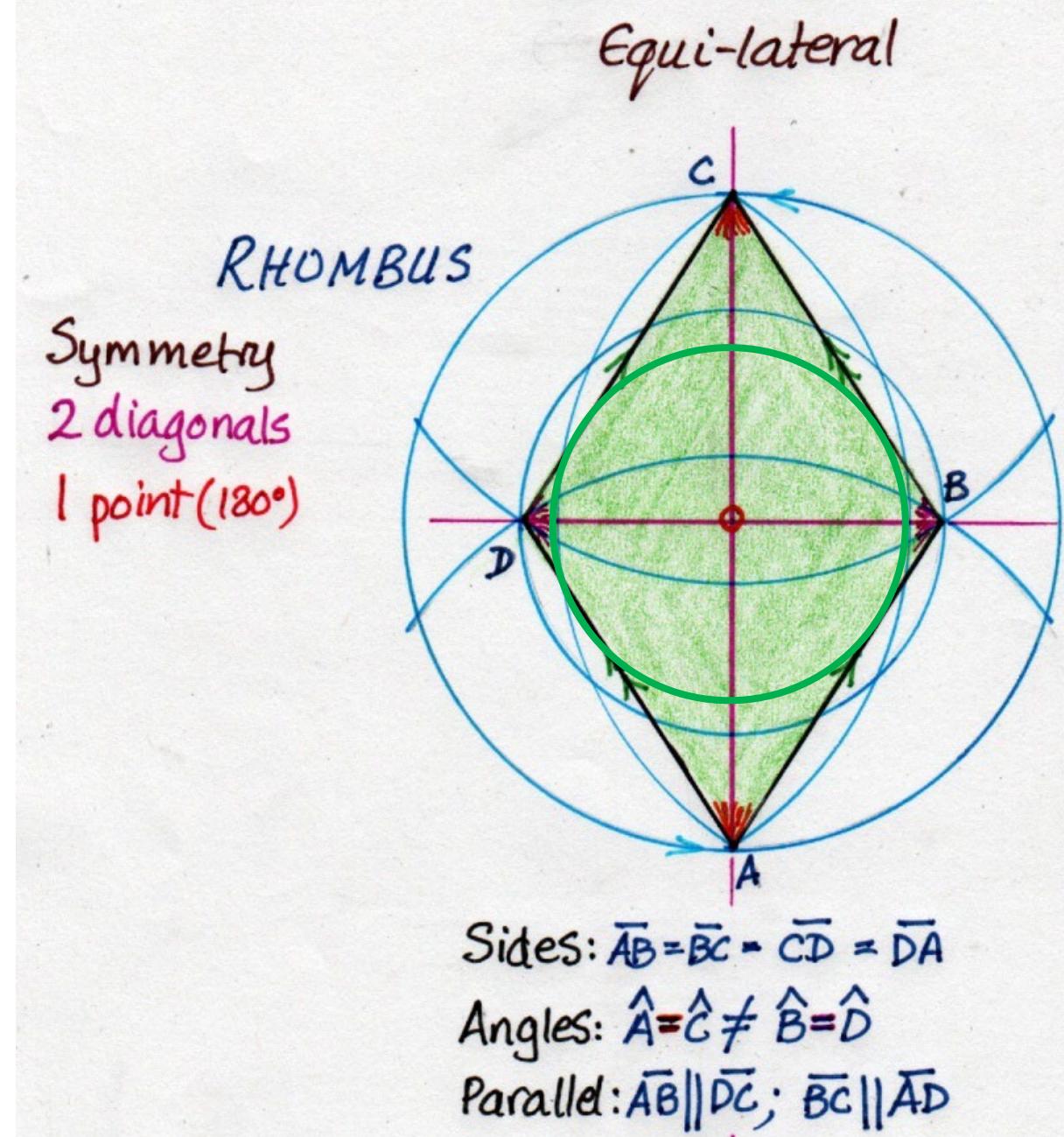
Oblong Rectangle

- 2 pairs of opposite sides equal.
- Equiangular: all angles equal = 90°
- 2 pairs of opposite sides parallel.
- 2 axes of symmetry: side bisectors
- 1 point of rotational symmetry (180°)
- **Circumcircle**

Equi-angular

RECTANGLE
Symmetry
2 bisectors
1 point (180°)

Sides: $\vec{AB} = \vec{DC} \neq \vec{BC} = \vec{AD}$
Angles: $\hat{A} = \hat{B} = \hat{C} = \hat{D} = 90°$
Parallel: $\vec{AB} \parallel \vec{DC}$; $\vec{BC} \parallel \vec{AD}$
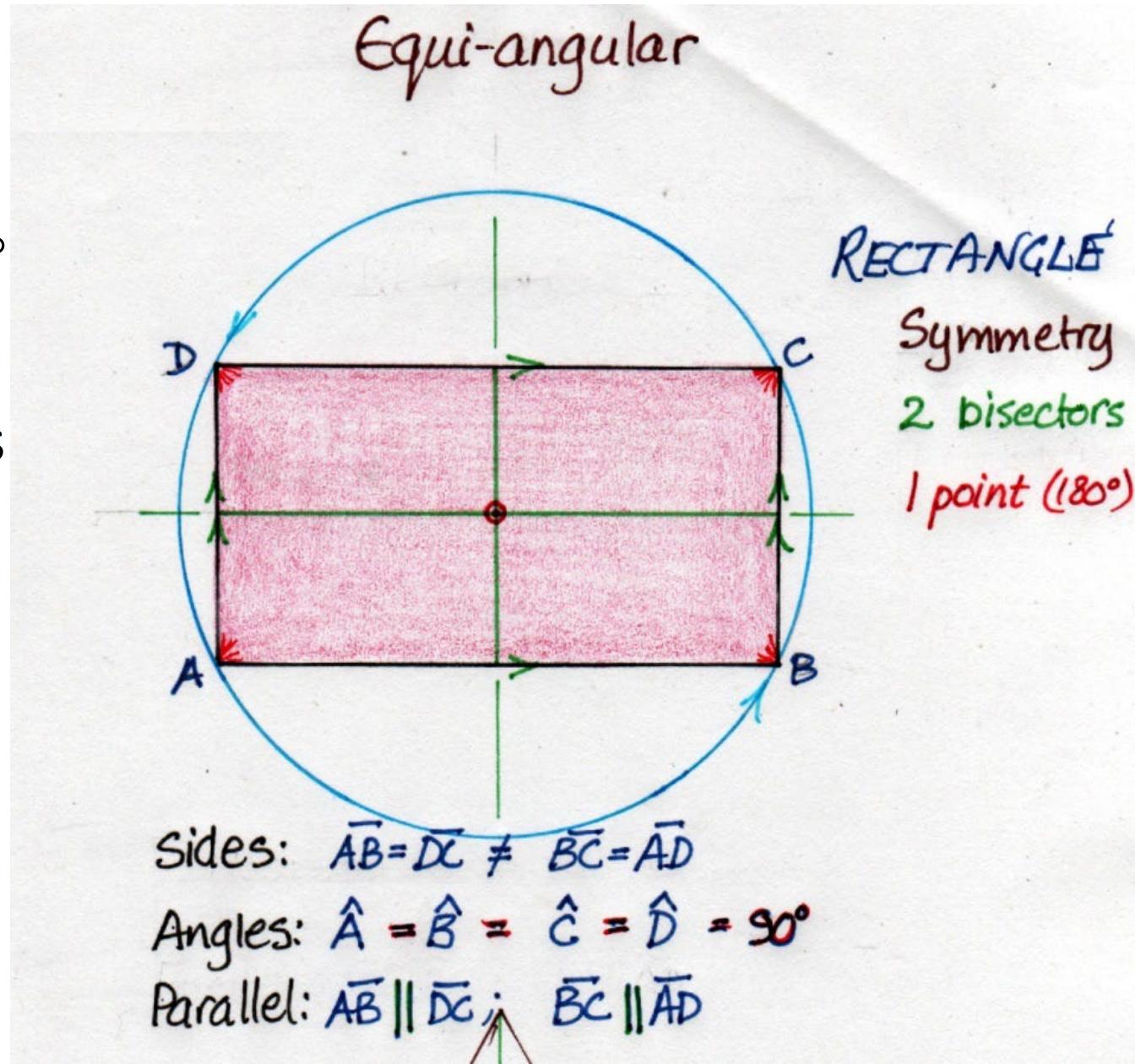
Parallelogram

- 2 pairs of opposite sides equal.
- 2 pairs of opposite angles equal.
- 2 pairs of opposite sides parallel.
- No axes of symmetry:.
- 1 point of rotational symmetry (180°)
- Neither Circumcircle nor Incircle.

PARALLELOGRAM

Sides $\overline{AB} = \overline{DC} \neq \overline{BC} = \overline{AD}$

Angles: $\hat{A} = \hat{C} \neq \hat{B} = \hat{D}$; $\hat{A} + \hat{B} = 180°$

Parallel: $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

Symmetry: 1 point (180°)
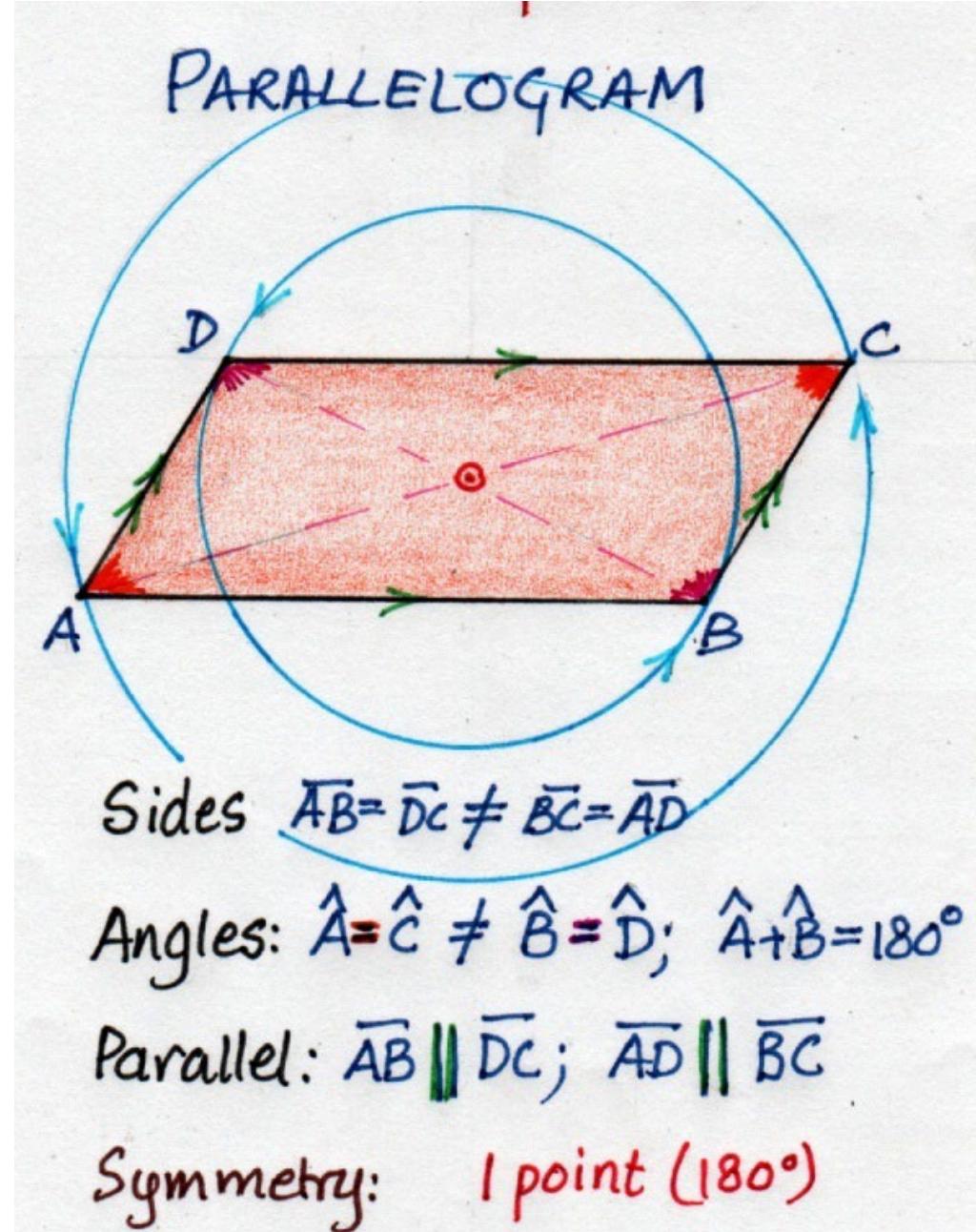
Square

- Equilateral: 4 sides equal.

- Equiangular: 4 angles equal

- 2 pairs of parallel sides

- 4 axes of symmetry: side bisectors and diagonals

- 1 point of rotational symmetry (90°))

- Both Circumcircle and Incircle.

Regular
Equi-lateral + Equi-angular
SQUARE

Sides: $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA}$
Angles: $\hat{A} = \hat{B} = \hat{C} = \hat{D} = 90°$
Parallel: $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$
Symmetry: 2 diagonals; 2 bisectors $^{\overline{AC}, \overline{BD}}$ ^{m,n}
1 point (90°)
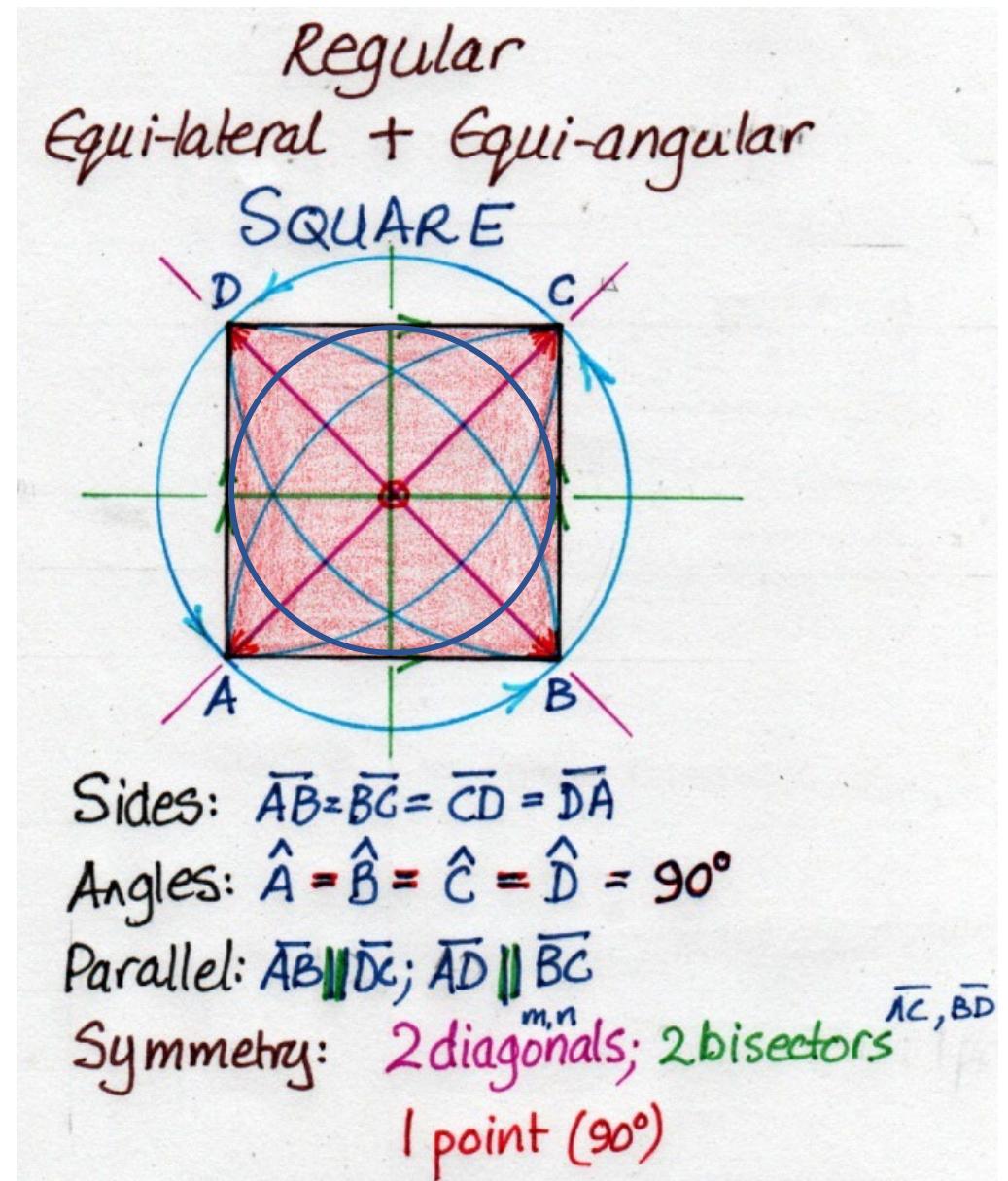
Cyclic quadrilateral

- All vertices lie on a circle.
- **Circumcircle**
- Opposite angles are supplementary [sum to 180°]
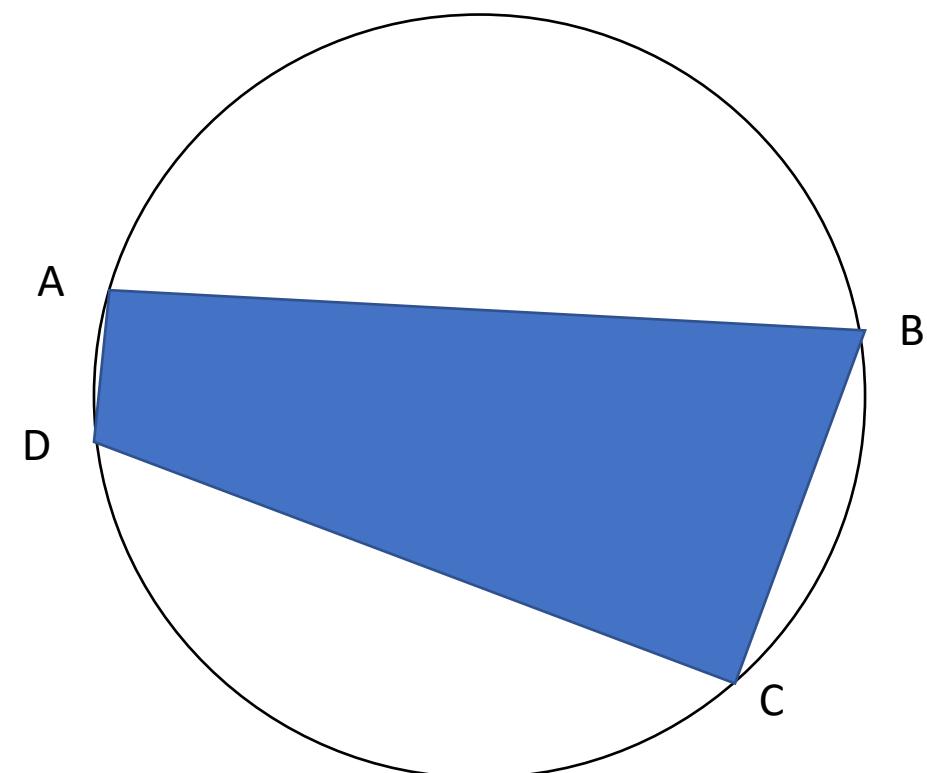
Tangential (escribed) Quadrilateral

- The two pairs of opposite sides in a tangential quadrilateral add up to the same total length.

- AB + CD = BC + CA = a+b+c+d = semi-perimeter
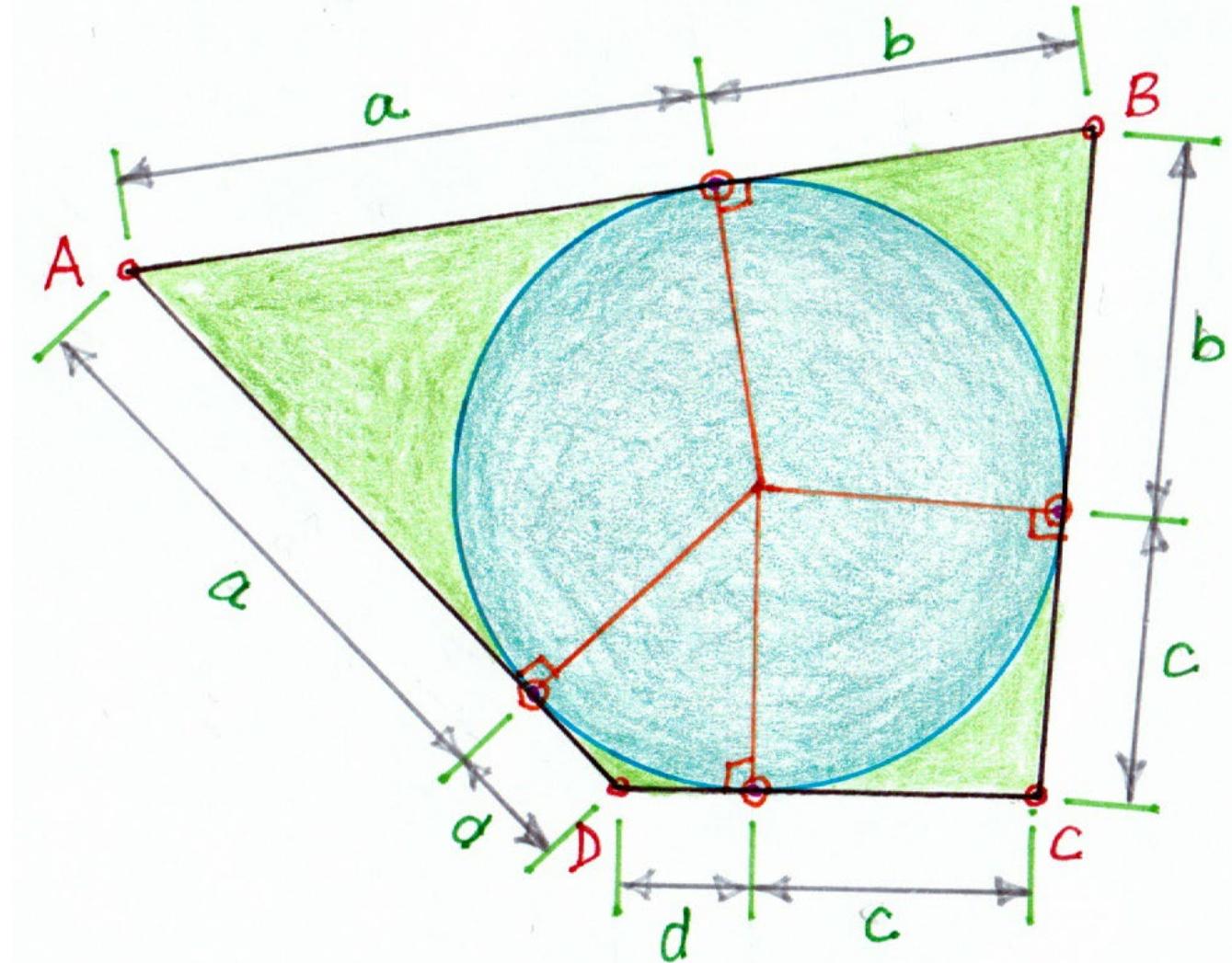
Multi-shape Tessellations.
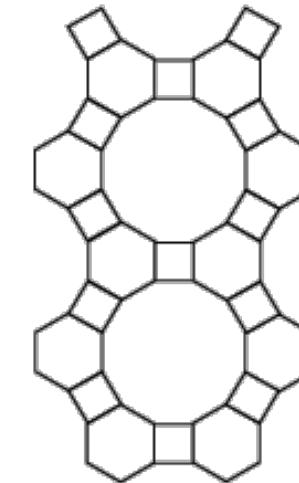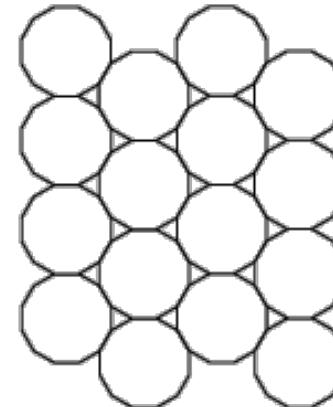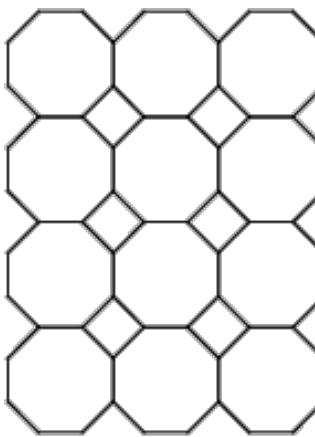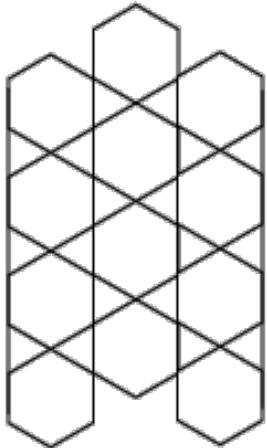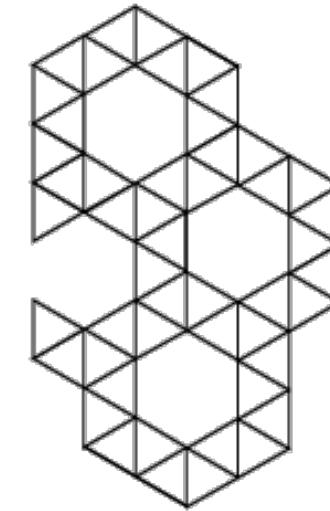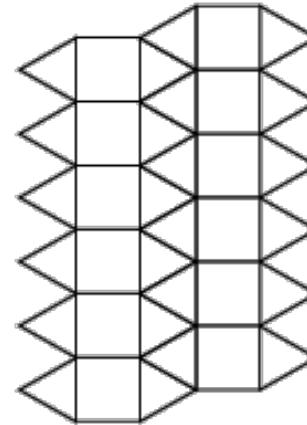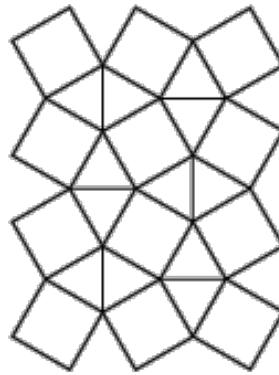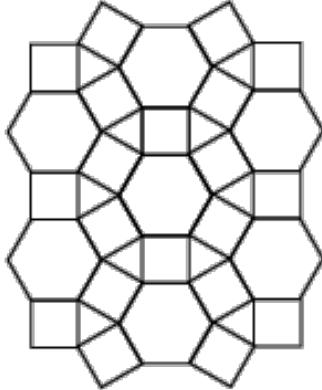
Single-shape Tessellations

- We can completely cover a surface with equal-sized regular polygons.
- **Only** regular **triangles**, **squares** or **hexagons** will cover a surface without gaps.
- Of these three, only squares have been adopted as a unit measure of area for a surface.

Regular Tessellations of a Plane

$6 \times 60° = 360°$

Triangles

$4 \times 90° = 360°$

Squares

$3 \times 120° = 360°$

Hexagons
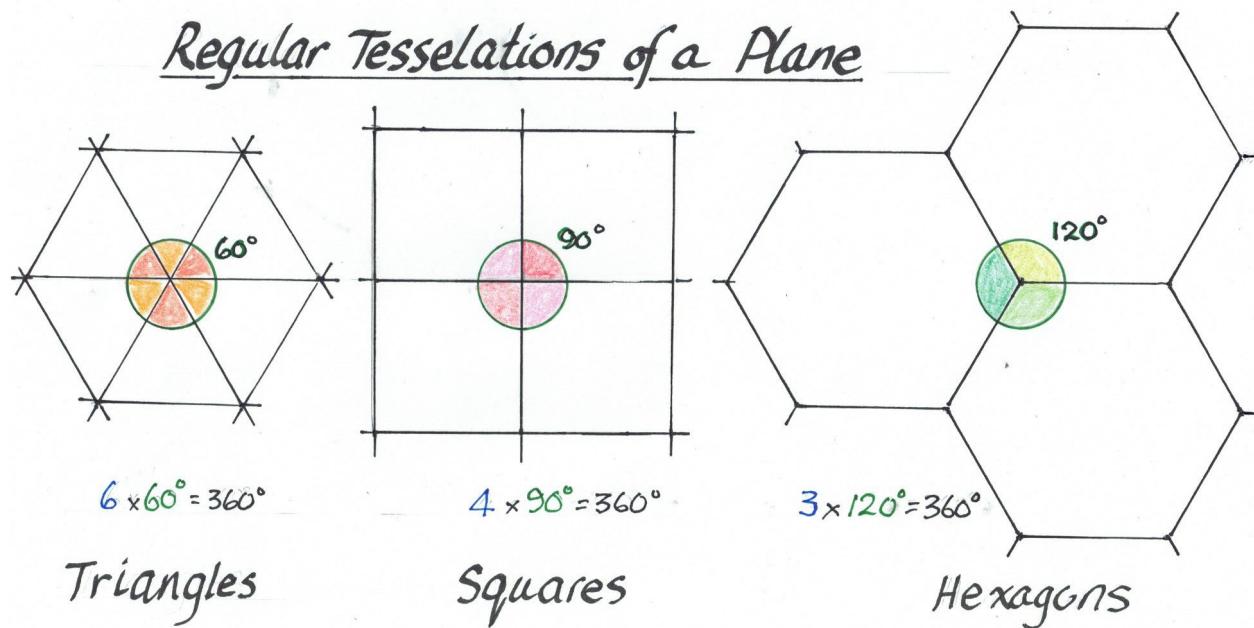
Area

- Area is a measure of the amount of surface within the boundary of a polygon or other plane figure.

- Our unit of measure needs to be uniform and consistent.

- A simple "practical" [hands-on] way is to use wheat or barley or rice grains in a tight, single layer over the surface within the bounds.

Using barley grains.

138 barley grains

70.7 mm
50 mm
50 mm

69 Barley grains

50 mm
50 mm
50 mm
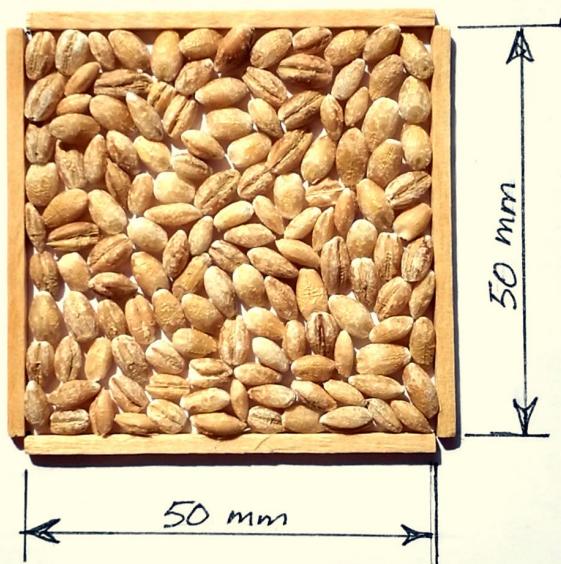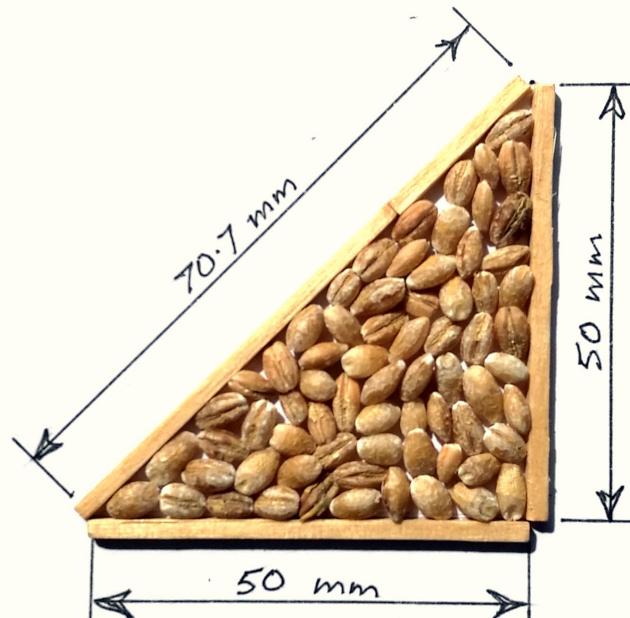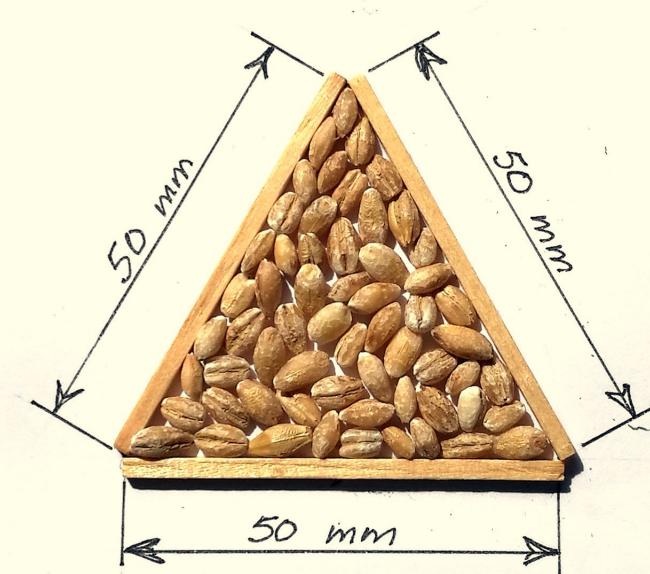
60 Barley grains

Area units

Length of side of square	Area of square
1 mm [millimetre]	1 sq. mm
1 cm [centimetre]	1 sq. cm = 100 sq. mm
1 m [metre]	1 sq. m = 10,000 sq. cm
10 m	1 are
100 m	1 hectare
1 km [kilometre]	1 sq. km = 100 hectare

Area – unit squares