

# Tessellations and Polyhedra

From 2 dimensions to 3 dimensions





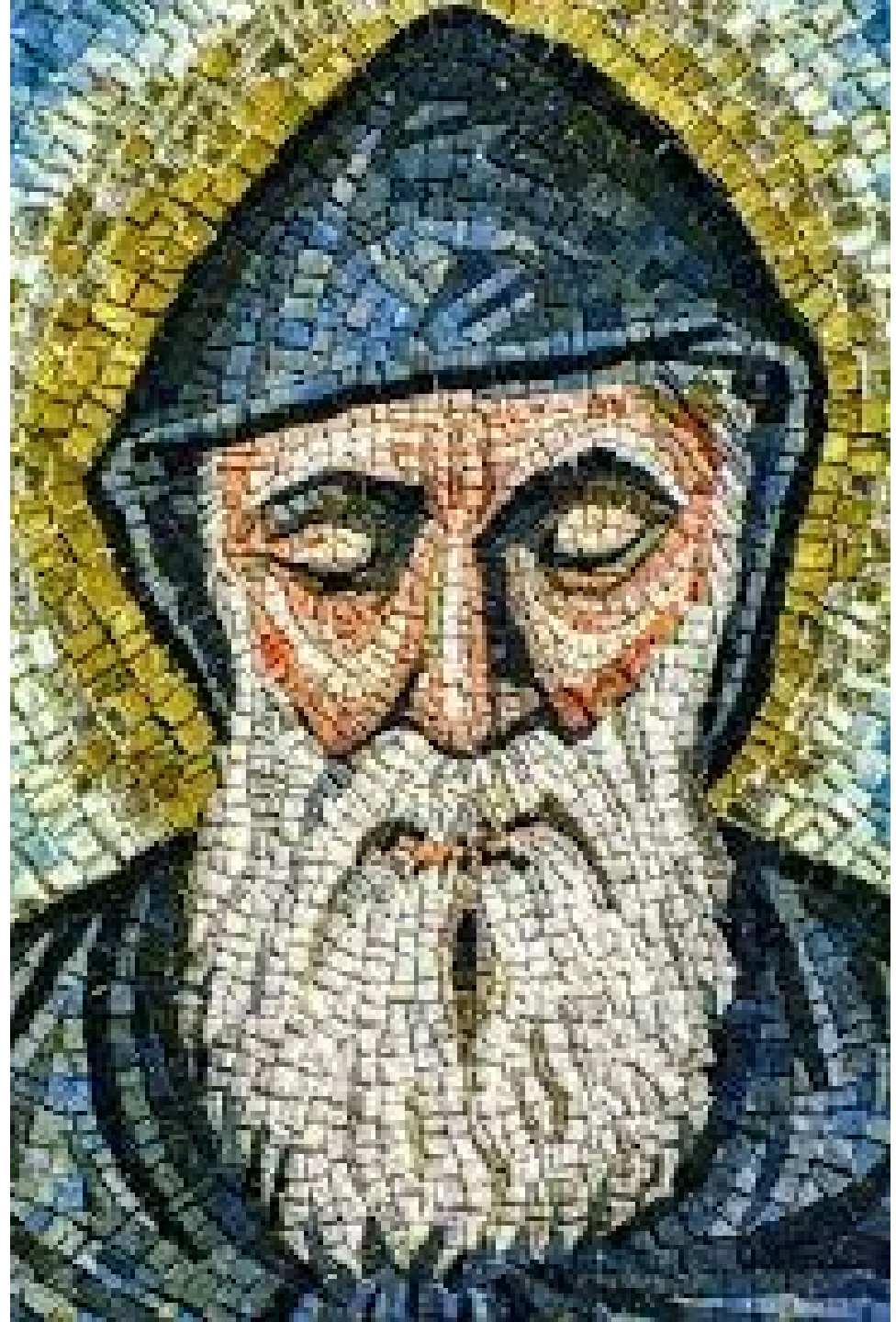
# Tessellation



# What is Tessellation?

- A Tessellation is an arrangement of shapes closely fitted together, especially of [polygons](#) in a repeated pattern without gaps or [overlapping](#).
- The name comes from Tessera [from Greek *tetra*, 4], the name for the small, coloured square tiles used in church or temple mosaics.
- You can find tessellations in floor tiling or carpet patterns, and in repeated pattern wallpaper.
- The artist M S C Escher used the idea to create intriguing interlocking patterns.

Mosaic art  
using small  
coloured tiles



# Rules of Tessellation

## **Rule #1**

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**The shapes must be regular polygons.**

## **Rule #2**

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The polygons can't overlap or have gaps in the pattern.

## **Rule #3**

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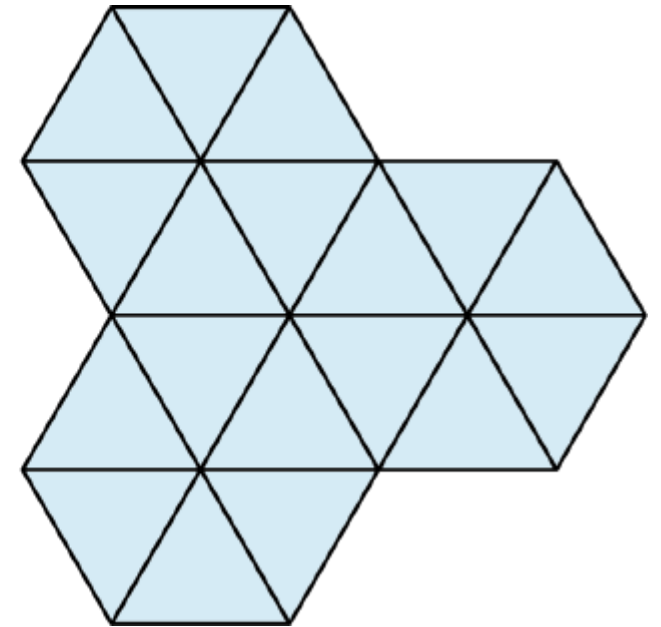
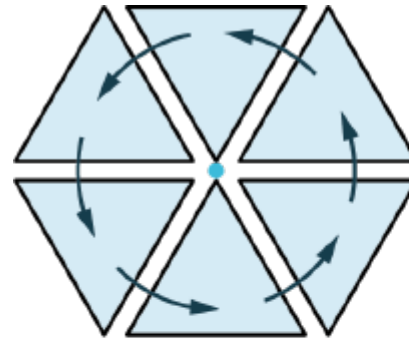
Every vertex has to look the same.

# Regular Tessellation

- A regular tessellation means that the pattern is made up of congruent regular polygons, same size and shape, including some type of movement; that is, some type of transformation or symmetry.

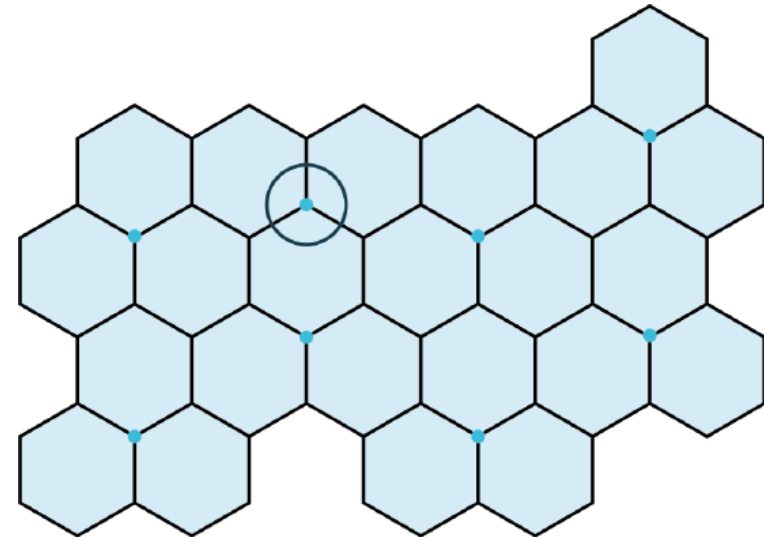
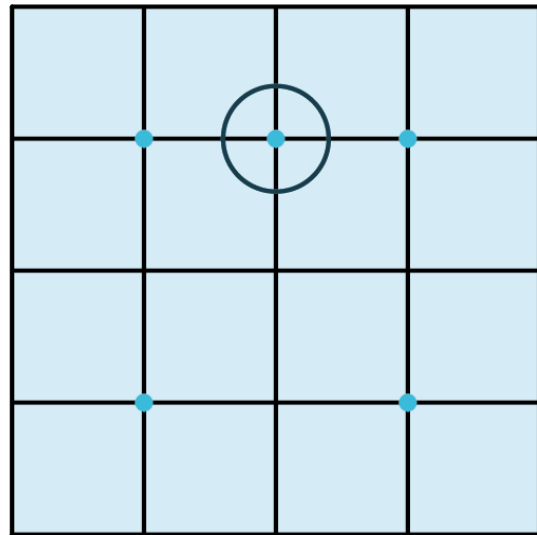
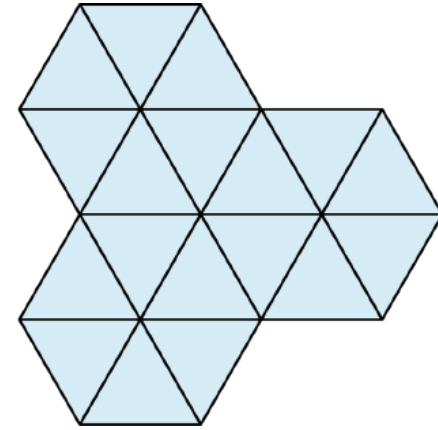
Here we consider the rigid motions of

- \* translation,
- \* **rotation**,
- \* reflection,
- \* glide reflection.



# Three basic patterns using regular polygons

- Meeting at, rotating around a point:
- 6 Trigons [triangles].  $6 \times 60^\circ = 360^\circ$
- 4 Tetragons [squares]  $4 \times 90^\circ = 360^\circ$
- 3 Hexagons  $3 \times 120^\circ = 360^\circ$

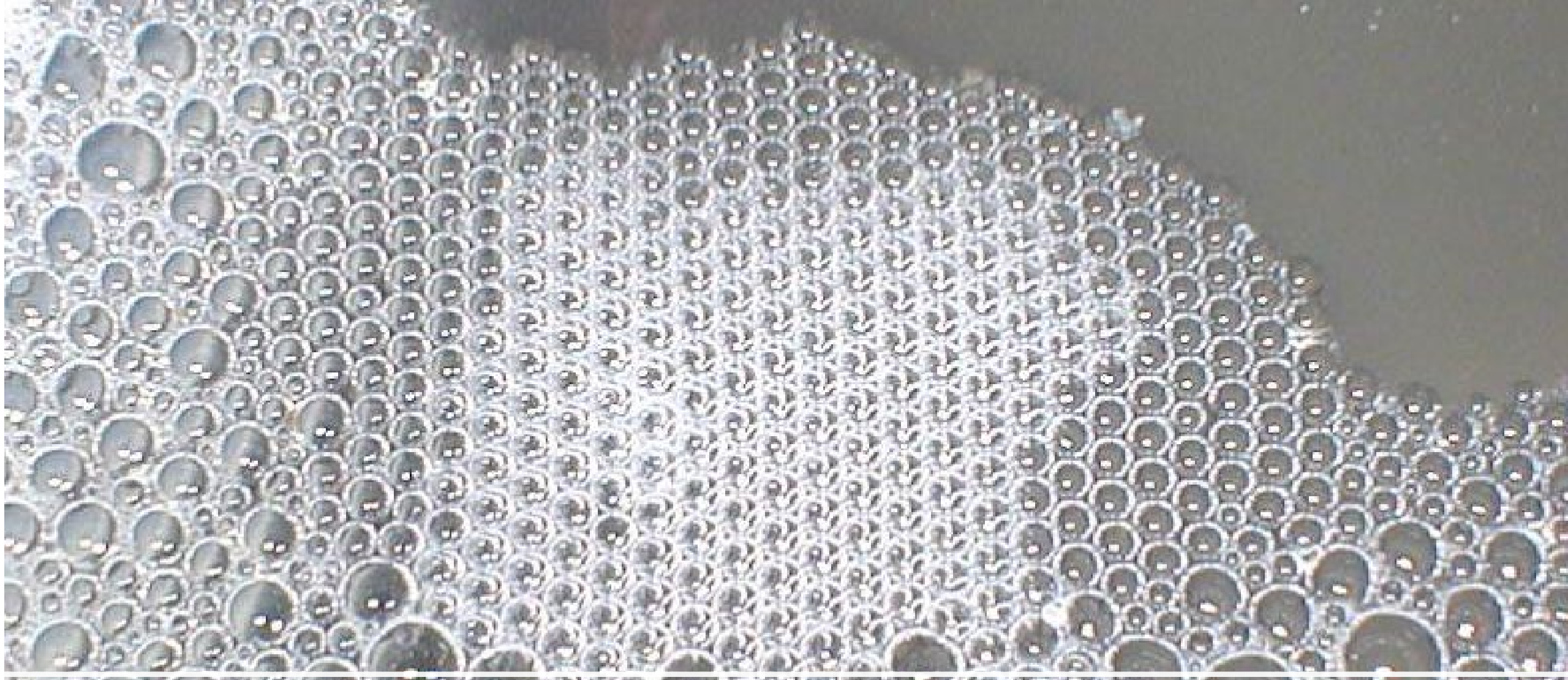


# Natural hexagon pattern [modified circles]

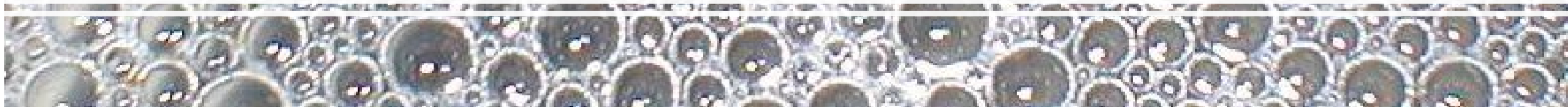


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Soap bubbles on water surface naturally form a hexagon pattern



# Escher: Birds flying both East and West

(basic pattern is Rectangular)



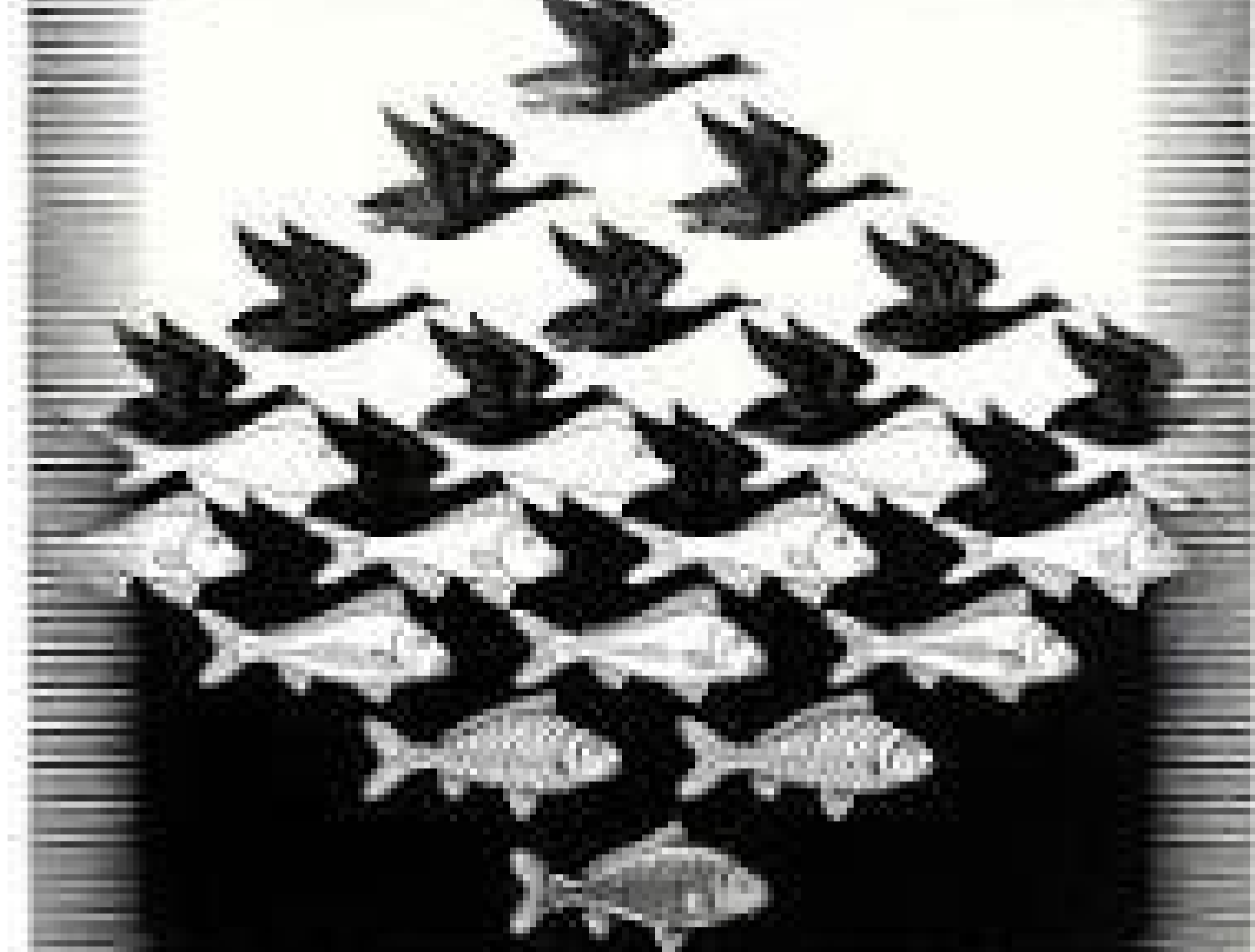
# Escher's lizards (basic pattern Triangular)

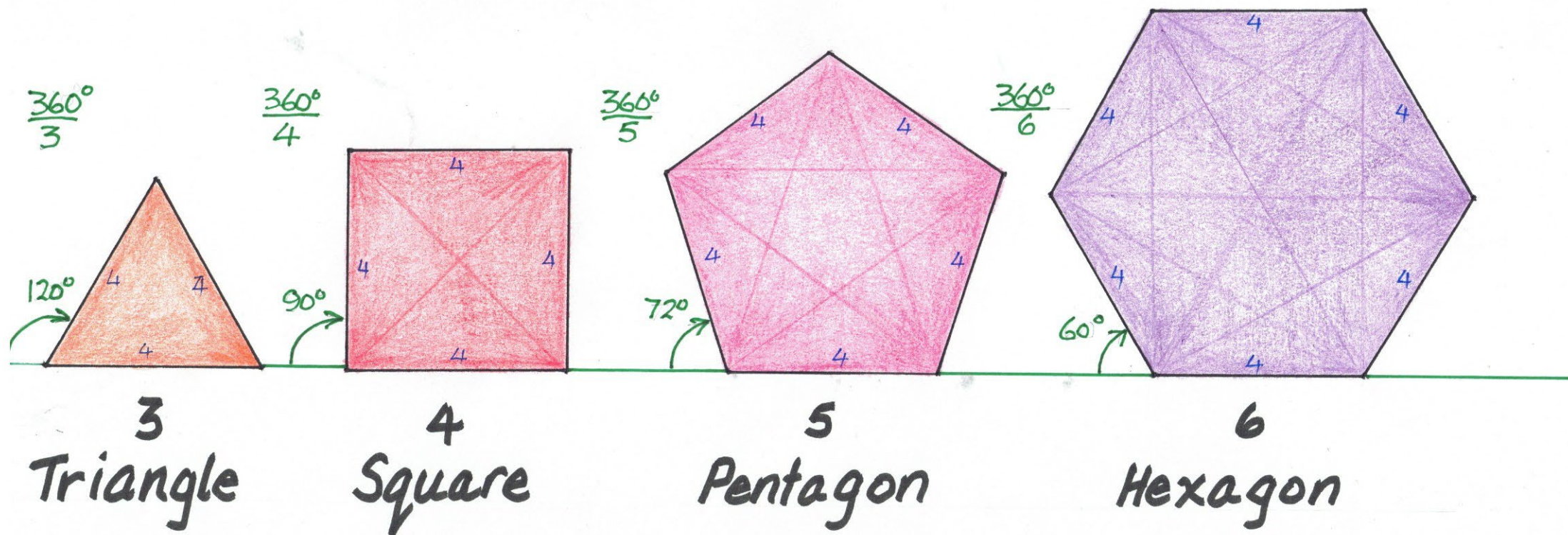


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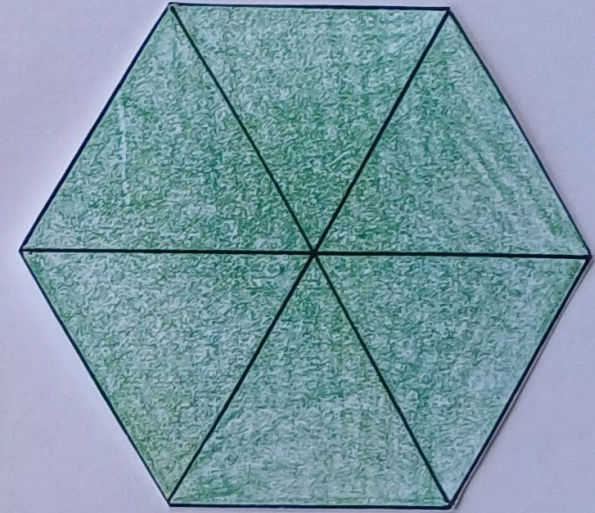
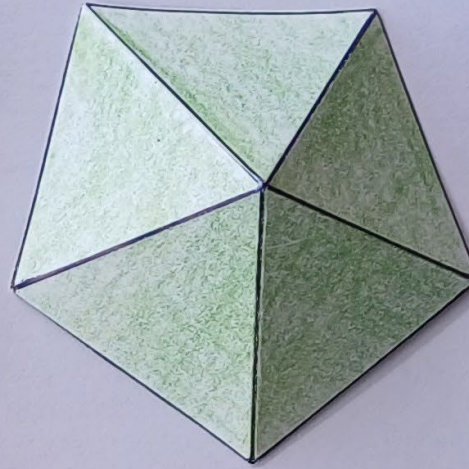
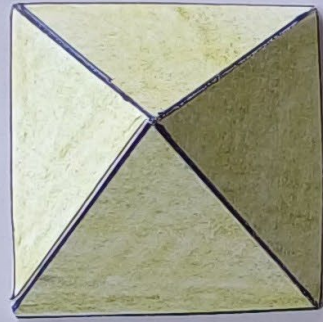
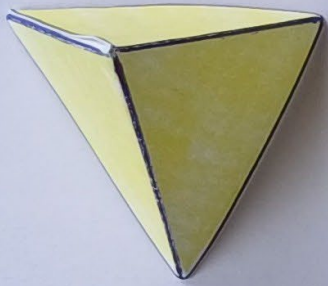
Escher  
artistically  
transforms  
birds into  
fish



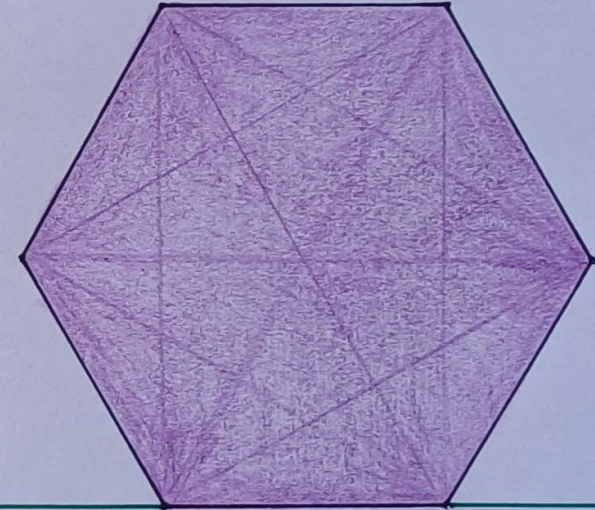
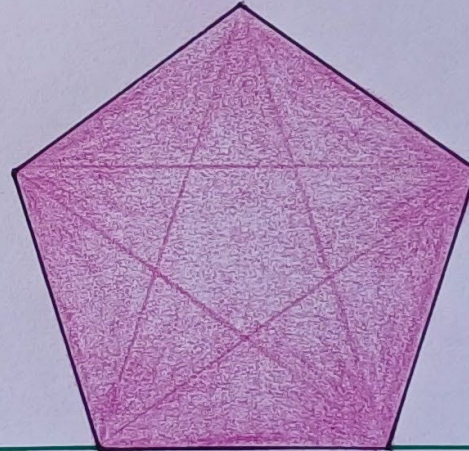
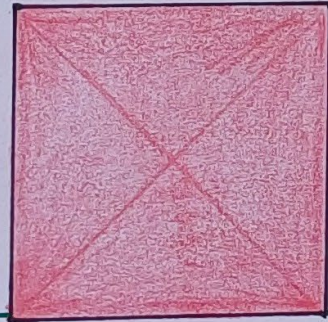
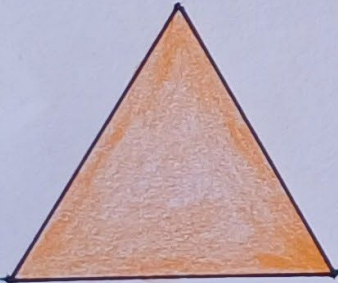


First four Regular [Equilateral + Equiangular] Polygons





Tetrahedron Octahedron Icosahedron



3  
Triangle

4  
Square

5  
Pentagon

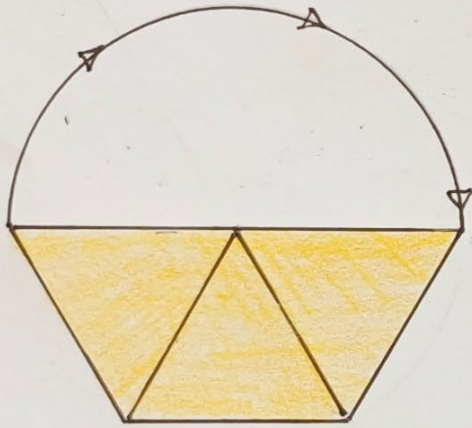
6  
Hexagon



# Patterns for folding 2-D into 3-D

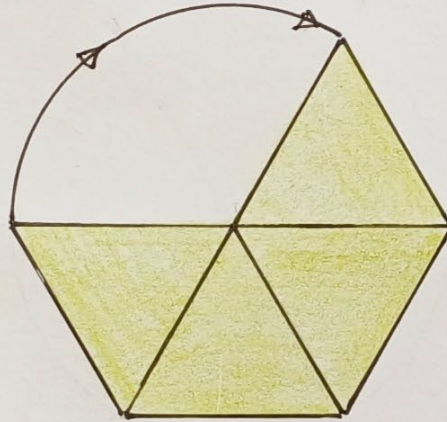
Sum of angles at vertex/point

$$3 \times 60^\circ = 180^\circ$$



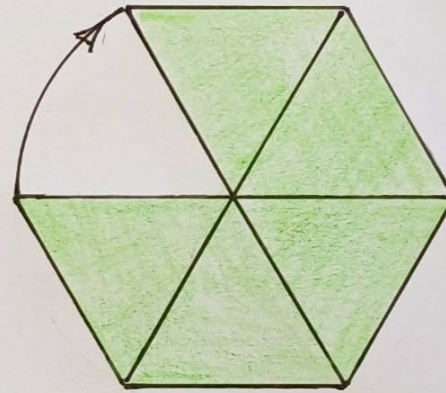
3

$$4 \times 60^\circ = 240^\circ$$



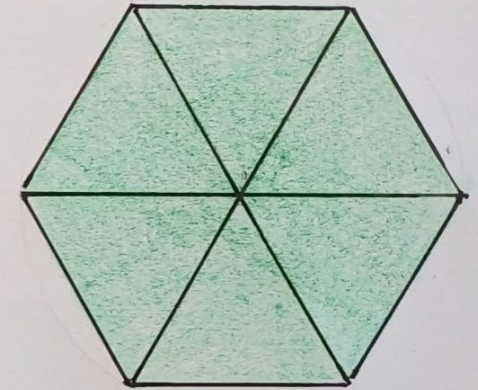
4

$$5 \times 60^\circ = 300^\circ$$



5

$$6 \times 60^\circ = 360^\circ$$



6

# Triangles meeting at a point

**Tetrahedron**

4 hedra

6 edges

4 vertex

**Octahedron**

8 hedra

12 edges

6 vertex

**Icosahedron**

20 hedra

30 edges

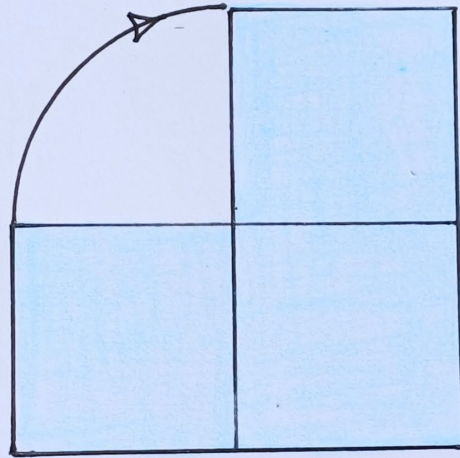
12 vertex

**Plane**

# Pattern to fold 2-D square into 3-D cubic form

Sum of angles at vertex/point

$$3 \times 90^\circ = \mathbf{270^\circ}$$



3

# Squares meeting at a point

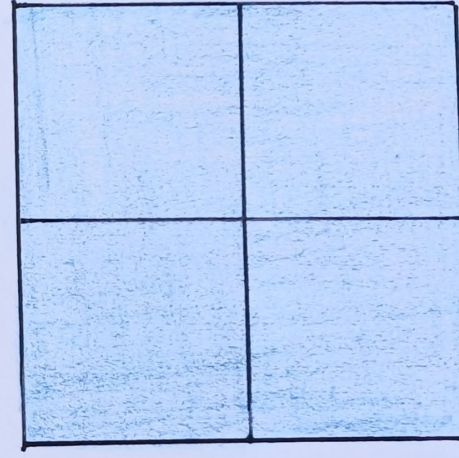
**Cube**

6 faces

12 edges

8 vertices

$$4 \times 90^\circ = 360^\circ$$



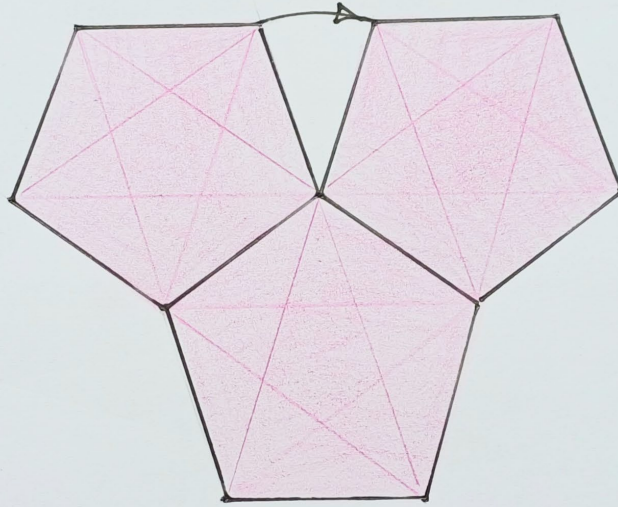
4

**Plane**

# Pattern for vertex of Dodecahedron

Sum of angles at vertex/point

$$3 \times 108^\circ = 324^\circ$$



*3 Pentagons meet at a point*

***Dodecahedron***

*12 hedra*

*30 edges*

*20 vertex*



# The five Platonic Solids are regular Polyhedra



# Plato

- The so-called Platonic solids are the five unique **convex, regular polyhedra** (tetrahedron, cube, octahedron, dodecahedron, and icosahedron) where all faces are identical, regular polygons, and the same number of faces meet at each vertex.  
Named for Plato, who, in his Socratic dialogue “Timaeus” related them to the Greek elements fire, earth, air, *zodiac*, and water, respectively.

Solid	Face Angle sum at vertex	# Faces	# edges	# vertex	Greek Element	Quality
Hexahedron	3 Square 270°	6	12	8	Earth rigid salt crystal	Matter Physical
Icosahedron	5 Triangle 300°	20	30	12	Water fluidity	Living Etheric
Octahedron	4 Triangle 240°	8	12	6	Air expansive	Conscious Astral
Tetrahedron	3 Triangle 180°	4	6	4	Fire radiant	Spirited Ego
Dodecahedron	3 Pentagon 324°	12	30	20	Universe Zodiac 12	Creative forces



For a dynamic view of the solids, see:

- <https://www.britannica.com/science/Platonic-solid>